MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXXVIII. MAY 11, 1928. No. 7

Rev. T. E. R. PHILLIPS, PRESIDENT, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Arthur T. Doodson, D.Sc., Tidal Institute, University of Liverpool ;
Sir Oliver Joseph Lodge, F.R.S., Normanton House, Lake, Salisbury ; and
Neil Henderson Smith, B.A., British Imperial Oil Co., Ltd., Melbourne, Australia.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Mervyn Archdall Ellison, The Observatory, Armagh, and 30 Trinity College, Dublin (proposed by T. H. E. C. Espin) ;
Michel S. Kovalenko, Ph.D., Assistant Professor of Mathematics and Astronomy, Swarthmore College, and Sproul Observatory, Swarthmore, Pa., U.S.A. (proposed by J. A. Miller) ; and
Arthur Douglas Merriman, M.A., B.Sc., A.I.Mech.E., 13 Grosvenor Drive, Whitley Bay, Northumberland (proposed by W. E. Curtis).

Seventy-five presents were announced as having been received since the last meeting, including amongst others :—

J. L. E. Dreyer, Tychonis Brahe, Opera Omnia, tomus xiv. (presented by Admiral Dreyer).
H. E. Purey-Cust, Sumner's Method, 2nd edition, 1928.
Astrographic Chart: Specola Vaticana, Zone $+55^\circ$ (32 charts); San Fernando Obs., Zone -3° (10 charts).

Spectrophotometry of the Chromosphere. By C. R. Davidson, M. Minnaert, L. S. Ornstein, and F. J. M. Stratton. (Plates 7, 8, 9.)

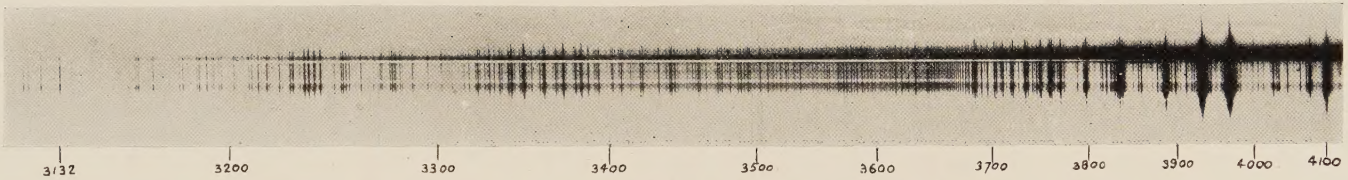
1. *Introduction.*—In the report of the British expedition to Sumatra to observe the total solar eclipse of 1926 January 14 (*Mem. R.A.S.*, **64**, 144, 1927), a few results were published which had been obtained by applying the Moll microphotometer to the spectrograms secured at the eclipse. The following paper deals with further results so obtained. It must be emphasised here that these results must not be regarded as final. For various reasons, connected with the eclipse conditions and due to the late stage in the preparations at which any spectrophotometry was included in the programme of observations, the necessary requirements for exact photometric work were not complied with. The intensity-density curves depend solely upon a plate of the solar spectrum taken with three exposures, of 1, 2 and 4 seconds, and developed independently of the eclipse plates. Under these conditions results cannot be relied upon as finally accurate ; they do, however, point to features of considerable interest which remain to be checked at some subsequent eclipse.

The value of Schwarzschild's coefficient p, in the formula $D = f(It^p)$ (connecting D the photographic density, I the intensity of the light, and t the time of exposure), is unknown for the film used, when developed in the damp tropical conditions which prevailed in Sumatra. A test film was examined later at Utrecht, and the value found for p was 0·8. Curves connecting the measured D and $\log(It^p)$ were worked out for $p = 1{\cdot}0$ and $p = 0{\cdot}8$, and the subsequent calculations were made in duplicate throughout. It may be stated at once that though there were naturally differences in the results obtained from the two curves, the general results were of the same order.

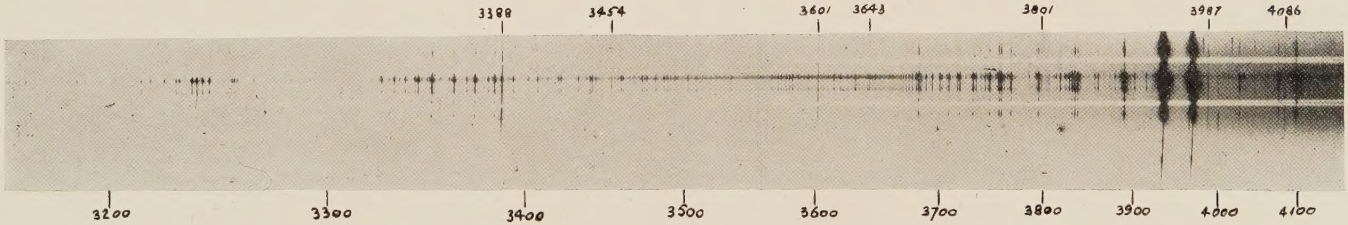
A further element of uncertainty underlying comparisons between different regions of the spectrum lies in the lack of complete knowledge of the distribution of radiation in the continuous spectrum of the Sun. The continuous spectrum on the comparison film has been compared with (*a*) Abbot's solar intensity curve for the energy distribution outside the atmosphere from his observations 1903–10 (*Smithsonian Institution Annals*, **4**, 203, 1922), (*b*) Abbot's similar curve from his later observations (*Smithsonian Miscellaneous Collections*, **74**, No. 7, p. 15, 1923), and (*c*) the black body curve for 6400° K, suggested by H. H. Plaskett's investigations (*Pub. D.A.O. Victoria*, **2**, 253, 1923). Many results have been worked out in triplicate. Of the six sets of results obtained from the two values of p and the three comparison curves used for sensitivity we shall take as our main set those given by $p = 0{\cdot}8$ with the black body curve for 6400° K used for the solar continuous spectrum. The degree of inaccuracy involved in the uncertainty in the value of p and the distribution of the solar intensity will be indicated by giving the different values obtained in a few selected cases. It may be noted here that the comparison between the photographed continuous spectrum of the Sun and the energy distribution in the solar spectrum outside our

Digitized by the Internet Archive
in 2026

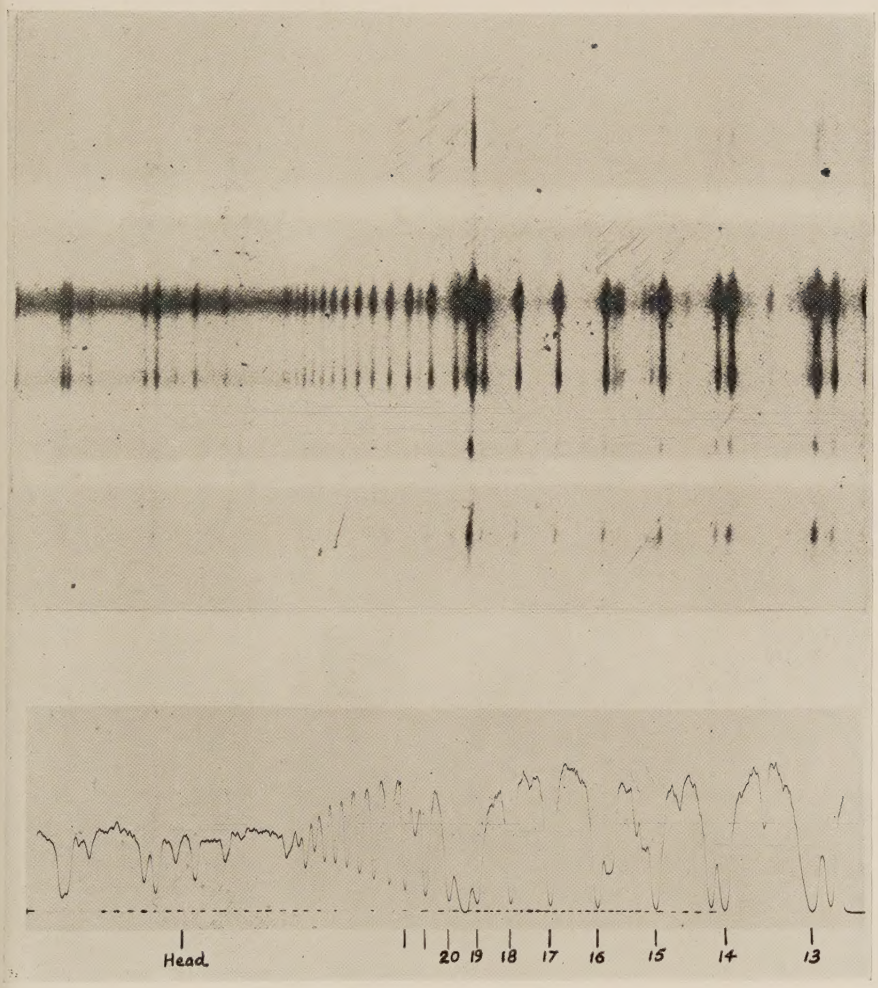
https://archive.org/details/monthly-notices-of-the-royal-astronomical-society_1928-05-11_88_7



Flash spectrum, photographed at Sumatra, 1926 January 14, by C. Davidson.



Spectra of corona and high prominence, photographed at Sumatra, 1926 January 14, by C. Davidson.



Balmer series in high prominence and photomicrometric trace.

atmosphere eliminates all complications arising from the sensitivity of the plate, the unequal transmission of the mirrors and the spectrograph, and the unequal transparency of the sky for different wave-lengths, so long as the comparison spectrum and the eclipse spectra are taken under exactly the same conditions. In the present case a fresh factor of uncertainty has been introduced owing to the comparison plate not being taken immediately after the eclipse.

2. *Intensities in Multiplets.*—The first application of the measured intensities of spectral lines was made to certain simple multiplets. It is, of course, impossible to say, *a priori* that the relative intensities in the chromosphere of lines in a multiplet will be the same as those found in the laboratory or indicated by the multiplet formulæ.* These formulæ confirmed in the laboratory, principally at Utrecht, refer to the emission from a very thin layer of gas. In the outer layers of the Sun the density is very small indeed, but on the contrary the layer of gas from which the flash spectrum is observed is extremely thick. It is therefore interesting to examine the measure of agreement found between the observed relative intensities and the formulæ, and to deduce from that some idea of the optical depth of the chromosphere. In the lower levels where the emitting source is viewed through considerable depths, with consequent weakening of the lines through self-reversal, and where the spectra become more complex and lines are more affected by blends, we may expect to find more divergence from the values given by the simple multiplet formulæ.

The results are given in tabular form, with added notes on the various multiplets examined. In some cases only a few lines of a multiplet are measured though the whole multiplet is present. The reason is that the other lines are either too bright or too faint for accurate measures of density, or else they are affected by close blends.

Notes on the successive multiplets are added, the numbers refer to the column 1 of the table.

1. The weaker line may be strengthened in the lower levels by blending with Hϵ. Taking Schwarzschild's coefficient for the film as 1·0 instead of 0·8 the measured ratio comes out at 1·80.

3. The weakest line here is close to a line which strengthens very much and blends with it at low levels. With the strongest line kept constant at 9 the weakest line increases from 0·9 to 3·8 as we pass from a high prominence down to the Baily's bead, from which the intensities published in the *Memoir* were obtained. For one of the intermediate levels in the Sun the photomicrometric trace shows this line clearly as a double, the second line being equally strong with the calcium line.

4. The stronger line is difficult to measure as it is close to the strong Sc^+ line at 3643. The given measure is the mean for two different heights.

7–9. The relative intensities in the chromosphere are much nearer

* L. de Kronig, *Zeits. f. Phys.*, **31**, 885, 1925; **33**, 261, 1925. Russell, *Proc. National Acad. of Science*, **11**, 314, 1925. Ornstein and Burger, *Zeits. f. Phys.*, **31**, 355, 1925. Sommerfield and Höhnl, *Sitzungsb. d. Preuss. Akad. d. Wiss.*, 1925, 141.

to the simple multiplet values than those found in the laboratory at Utrecht by Mr. Bouma—from a theoretical point of view for complicated spectra, such as that of nickel, the simple multiplet rules may fail, while probably the sum rule must also be extended in these cases. A paper by Ornstein and Bouma on these points will shortly appear in the *Zeitschrift für Physik.*

9. The line at 3515·06 strengthens markedly at lower levels. Keeping 3414·77 constant at 15 it rises from 14·2 to 19·5. Possibly some blend is involved.

TABLE I.

Multiplet Intensities.

Element.	Wave-length.	Multiplet.	Relative Intensities. (*a*) According to Multiplet Formulæ.	(*b*) Observed.	(*c*) Observed in Laboratory.	Height above Limb of Sun.
1. *Ca*+	3933·66	$1^2S_1-1^2P_2$	2	1·75	..	Mean for 8 different heights.
	3968·47	$1^2S_1-1^2P_1$	1	1·0	..	
2. *Ca*+	3706·02	$2^2P_2-1^2S_1$	2	2·2	..	0·031*r*
	3736·90	$2^2P_1-1^2S_1$	1	1·0		
3. *Ca*+	3158·88	$2^2D_2-1^2P_1$	5	4·4	..	0·020*r*
	3179·34	$2^2D_3-1^2P_2$	9	9		
	3181·28	$2^2D_2-1^2P_2$	0·5	0·9		
4. *Ti*+	3624·84	$1^2S_1'-1^2P_1'$	1	1·0	..	0·020*r*
	3641·33	$1^2S_1'-1^2P_2'$	2	1·74		
5. *Ti*+	3504·89	$2^2G_5-2^2G_5'$	10	10·0	..	0·020*r*
	3510·84	$2^2G_4-2^2G_4'$	8	8·8		
6. *Ti*+	3308·79	$2^4F_4'-1^4F_5$	2·0	0·6	..	0·020*r*
	3318·01	$2^4F_3'-1^4F_4$	2·5	1·5		
	3326·76	$2^4F_2'-1^4F_3$	1·9	1·8		
	3329·45	$2^4F_4'-1^4F_4$	9·0	9·0		
7. *Ni*	3413·99	$1^3D_2'-1^3F_2'$	1·0	1·0	33	0·020*r*
	3513·95	$1^3D_1'-1^3F_2'$	7·8	8·4	22·3	
8. *Ni*	3524·52	$1^3D_3'-1^3P_2'$	5·6	4·5	100	0·020*r*
	3597·76	$1^3D_1'-1^3P_1'$	1·0	1·0	66	
9. *Ni*	3414·77	$1^3D_3'-1a^3F_4'$	15	15	92·5	0·020*r*
	3458·47	$1^3D_1'-1a^3F_2'$	7·8	8·5	91·5	
	3515·06	$1^3D_2'-1a^3F_3'$	10·4	14·2	85·5	

Table I. indicates for the most part reasonable agreement between the measured intensities of bright lines in the spectrum of the high

chromosphere or of a prominence and the relative intensities of lines given by the multiplet formulæ. The indication of the result is that the high chromosphere is a layer of small optical depth.

As an indication of the effect on the relative intensities of changing the value of Schwarzschild's coefficient p and of using different intensity curves of the solar spectrum, from which to derive the sensitivity of the film, we give below alternative figures for the observed relative intensities of the lines in multiplets 8, 9.

		Plaskett.	Abbot, 1922.	Abbot, 1923.
8.	$p=0{\cdot}8$	4·5 : 1	4·2 : 1	4·3 : 1
	$p=1{\cdot}0$	5·2 : 1	4·8 : 1	
9.	$p=0{\cdot}8$	15 : 8·5 : 14·2	15 : 9·2 : 16·5	15 : 8·9 : 15·7
	$p=1{\cdot}0$	15 : 7·8 : 14·2	15 : 8·4 : 16·6	

We may note that Plaskett's curve and $p = 1{\cdot}0$ gives the closest agreement with the multiplet formulæ.

3. *The Balmer Series.*—One of the most interesting applications of the microphotometer to the eclipse spectra lies in a study of the Balmer series in the chromospheric spectrum. The formula that we have tested is Schrödinger and Pauli's formula (*Ann. d. Phys.*, **80**, 490, 1926), which gives for a line of series number l (for Hα, $l = 3$) the intensity I in the form

$$\left.\begin{aligned} \mathrm{I}_l &= \frac{4^3(l-2)^{2l-3}(3l^2-4)(5l^2-4)}{l(l+2)^{2l+3}} e^{\frac{h\mathrm{R}}{l^3kT}} \\ &= \phi(l) e^{\frac{h\mathrm{R}}{l^3kT}} \end{aligned}\right\} \quad . \quad . \quad (1)$$

where
$h = 6{\cdot}55 \times 10^{-27}$ = Planck's constant,
$\mathrm{R} = 3{\cdot}29 \times 10^{15}$ = Rydberg's constant,
$k = 1{\cdot}37 \times 10^{-16}$ = Boltzmann's constant,
T = absolute temperature.

The relative intensities in the eclipse spectrograms (see Plate 8) are found to be much more closely in agreement with this formula than in any previous determination from laboratory spectra. The lines fall off in intensity slightly more rapidly than the formula indicates, but nothing like as rapidly as in the laboratory.*

The falling off of intensities along the series is generally much stronger in the laboratory than is given by the formulæ of quantum mechanics. If self-reversal is effective in the high chromosphere the tendency would be to lessen the steep slope of the intensity curve found in the laboratory, and the closer agreement found with the Schrödinger-Pauli formula may be due to some such cause. On the other hand, it may be of interest to assume that the observation ought to show agreement with the formula and to examine the cause for the slight divergence found.

* *Cf.* I. A. Bongers, *Intensiteits-metingen in de Balmerreeks, Utrecht Dissertation*, 1927.

It is natural to examine first the effect of the uncertainty as to the Schwarzschild coefficient p and the sensitivity of the plate. Reductions with $p = 1{\cdot}0$ and with Abbot's curve for the solar continuous spectrum only tend to a slight increase in the divergence under examination. The divergence from the formula was found to increase slightly with l, the term number in the series, and it was felt that this might be due to the fact that owing to limitation to lines of measurable photographic density, the lines nearer the head of the series had had to be examined in lower levels of the chromosphere. The pressure effect discussed by Urey,* R. H. Fowler, and Fermi at once suggested itself as a likely cause for weakening the fainter lines of the series. An examination shows, however, that this cause when it becomes effective acts much too rapidly down the series to correspond to the gradual effect observed, which requires some other explanation. The effect of the more intense radiation from the lower levels in ionizing more readily the atoms with larger orbits has been examined by Mr. W. H. M'Crea, and he informs us that he finds no explanation there of the observed divergence.

An element of uncertainty is introduced into the intensities of lines beyond $l = 22$ by the strengthening of a background of apparently continuous spectrum between the last lines of the series distinctly observed ; † this is probably due to the crowding together of the lines of the Balmer series and the superposition of their weaker wings. No reasonable estimate of the effect of this background seems able to account for the falling off of intensity in the series, which is clearly marked before the lines are sufficiently close for the background to be measurable.

The temperature factor $e^{\frac{hR}{l^2kT}}$ involved in the formula acts in the right sense to account for the divergence, but it leads to impossibly low values of T if the observed divergence from the formula for high values of l is to be explained as due to temperature. Thus, comparing the lines $l = 10$, $l = 16$ at a height of 20,000 km. above the Sun's limb we get $T = 2600°$ K, while $l = 23$ and $l = 27$ at a height of 700 km. gives $T = 150°$ K. It is obvious that the variation of $I_l/\phi(l)$ along the series is not to be attributed to temperature, and that temperature determinations are illusory. The only one secured that seems reasonable to us is one from a comparison of the lines $l = 6$, $l = 9$, in a prominence at a height of 30,000 km., which gives $T = 4750°$ K.

From measures made on different groups of lines of the Balmer series at different heights we have worked out the relative intensities of the lines and compared them with the Schrödinger-Pauli formula for several different temperatures. In Table II. we give the relative values for selected pairs of lines of

$$I'_l = \frac{I_l e^{-\frac{hR}{l^2kT}}}{\phi(l)},$$

where I_l is the measured brightness of line l and $\phi(l)$ is given by equation

* *Ap. J.*, **59**, 1, 1924.

† Compare "Absorption in Stellar Spectra," *L.O.B.*, **12**, 155, 1926.

(1) above. These values have been determined for T = 5000° K and T = 4000° K. If the formula agreed with observation I'_{l_1}/I'_{l_2} would be equal to unity. Assuming that I'_l/I'_{l+1} is constant throughout each range, we have given in the last columns of Table II. the values of I'_l/I'_{l+1} for each range considered. The height given in column 3 is the height above the Sun's limb to which the measures refer, measured as a fraction of the Sun's radius. In the greater heights taken from the coronal photographs, where the position of the slit was not kept constant referred to the Sun's limb, the height refers to the lowest level reached during the whole exposure at the point of the slit where the measures have been made.

TABLE II.

Relative Intensities of Balmer Lines.

			I'_{l_1}/I'_{l_2}.		I'_l/I'_{l+1}.	
l_1.	l_2.	h.	T=4000°.	T=5000°.	T=4000°.	T=5000°.
6	9	0·044r	0·91	1·03	0·97	1·01
10	16	0·031r	1·14	1·20	1·02	1·03
12	17	0·0066r	1·21	1·24	1·04	1·04
16	23	0·020r	1·20	1·21	1·03	1·03
21	28	0·020r	1·71	1·73	1·08	1·08
15	28	0·020r	2·04	2·07	1·06	1·06
21	27	0·022r	2·01	2·03	1·12	1·13
23	27	0·001r	1·71	1·72	1·14	1·15

An examination of the above table suggests that depth in the solar atmosphere is the chief factor in causing divergence from the Schrödinger-Pauli formula. It may be of interest to give a complete series of values line by line for the longest set measured at one height, namely $l = 15$ to $l = 28$, examined in the prominence on the coronal plate at a height of 14,000 km. above the Sun's limb. The temperature 5000° K is assumed and the quantity tabulated is

$$\log_e \frac{I_l}{\phi_l} - \frac{hR}{l^2kT}$$

which should be constant if the intensities of the lines agreed with the formula.

TABLE III.

Balmer Series—Comparison with the Schrödinger-Pauli Formula.

Line. l.	$\log_e \frac{I_l}{\phi_l} - \frac{hR}{l^2kT}$.	Line. l.	$\log_e \frac{I_l}{\phi_l} - \frac{hR}{l^2kT}$.
15	15·50	22	15·22
16	15·43	23	15·23
17	15·41	24	15·13
18	15·27	25	15·03
19, 20	Blends	26	15·06
		27	14·82
21	15·31	28	14·77

4. *Lines of an Ionized Atom.*—The relative brightness of the lines of an ionized element at different levels may be used to determine the difference of temperature between these levels. Ionized titanium is present strongly both in the flash and in the high prominence, and it is not unreasonable to assume that the fraction of the atoms in the different stages of excitation would not differ much from that given by a Boltzmann distribution. The intensity of a line is proportional to the total concentration of the atoms, the fraction in the excited state considered and the emission probability. For two lines in the flash, the intensities I_A, I_B are given by

$$I_A = Cg_A e^{-\frac{E_A}{kT}}\epsilon_A$$

$$I_B = Cg_B e^{-\frac{E_B}{kT}}\epsilon_B,$$

where C is the concentration of the titanium ions, T is the temperature in the flash layer, E_A E_B the energy differences between the higher levels involved in the line and the lowest level in the titanium ion, g_A, g_B the statistical weights, and ϵ_A, ϵ_B the emission coefficients of the two levels. For the same lines in the prominence of temperature T′ the intensities are given by

$$I'_A = C'g_A e^{-\frac{E_A}{kT'}}\epsilon_A$$

$$I'_B = C'g_B e^{-\frac{E_B}{kT'}}\epsilon_B$$

This gives the relation

$$\frac{I_A}{I'_A} = \frac{C}{C'}\frac{e^{-\frac{E_A}{kT}}}{e^{-\frac{E_A}{kT'}}}$$

$$\frac{I_B}{I'_B} = \frac{C}{C'}\frac{e^{-\frac{E_B}{kT}}}{e^{-\frac{E_B}{kT'}}}$$

or

$$\frac{I_A}{I'_A}e^{\frac{E_A}{k}\left(\frac{1}{T}-\frac{1}{T'}\right)} = \frac{I_B}{I'_B}e^{\frac{E_B}{k}\left(\frac{1}{T}-\frac{1}{T'}\right)}$$

$$= \text{same constant quantity for each line.}$$

If we express the relative energies of the levels in wave numbers ν,

$$\frac{E}{k} = \frac{h\nu c}{k} = 1{\cdot}43\nu;$$

we can obtain the values of ν direct from the Tables I. and II. of Russell's paper on the spectrum of ionized titanium (*Ap. J.*, **64**, 283, 1927).

Plotting $\log_e \frac{I}{I'}$ against $1{\cdot}43\nu$ for a number of lines we can get the best value of $\left(\frac{1}{T} - \frac{1}{T'}\right)$, which fits the observations.

Table IV. gives for a number of lines of ionized titanium of measurable density in the flash at a height of 700 km. and in a prominence at a height of 14,000 km. the observed values of I/I′ and also the values of $1{\cdot}43\nu$.

TABLE IV.

Lines of Ionized Titanium at Different Heights.

Wave-length.	Multiplet Definition.	$1{\cdot}43\nu$.	I/I′.
3394·57	$a^4F_3'-a^4G_3'$	$4{\cdot}22\times10^4$	3·03
3444·31	$b^4F_5'-a^4G_6'$	4·33	3·01
3318·01	$b^4F_3'-a^4F_4$	4·45	3·58
3308·79	$b^4F_4'-a^4F_5$	4·48	4·60
3624·84	$a^2P_1'-a^2S_1'$	5·35	5·82
3332·10	$b^4P_2'-a^4S_2'$	5·72	5·35
3315·32	$b^4P_1'-a^4S_2'$	5·72	6·93
3510·84	$b^2G_4-b^2G_4'$	6·26	6·79
3504·89	$b^2G_5-b^2G_4'$	6·26	5·49
3456·40	$b^2P_2'-b^2P_2$	6·51	8·02

The graph gives

$$\frac{I}{T'} - \frac{I}{T} = \frac{I}{27400}.$$

If we put T = 5000° K this gives T′ = 4200° K. The difference of temperature between the two levels does not seem unreasonable. This method probably gives the most accurate measure of the temperature as yet available for the high chromosphere.

5. *The Continuous Spectrum of the Lower Corona.*—The continuous spectrum arising either from the corona itself or more probably from a high prominence (which crossed the slit for part of the exposure made with the quartz spectrograph for the coronal spectrum) has also been studied at different heights and compared with black body curves

$$\left(I = \frac{c}{\lambda^5} e^{-\frac{1{\cdot}43}{\lambda T}}\right)$$

to see what temperature is indicated (see Plate 9). The spectrum is only of measurable density over the range 4100–3850 A, and for this range it has been examined at four different heights. The value of h in the following tables refers to the lowest level of the Sun's atmosphere reached at the point of the slit where the measures were made; as already stated in photographing the coronal spectrum to get as much low coronal radiation as possible, the slit was made to follow the Moon's limb during the exposure so that the observed density at any point of

the slit is in two senses an integrated effect. Values are given for $p = 0{\cdot}8$ and $p = 1{\cdot}0$, also for Plaskett's and Abbot's reduction curves for sensitivity.

TABLE V.

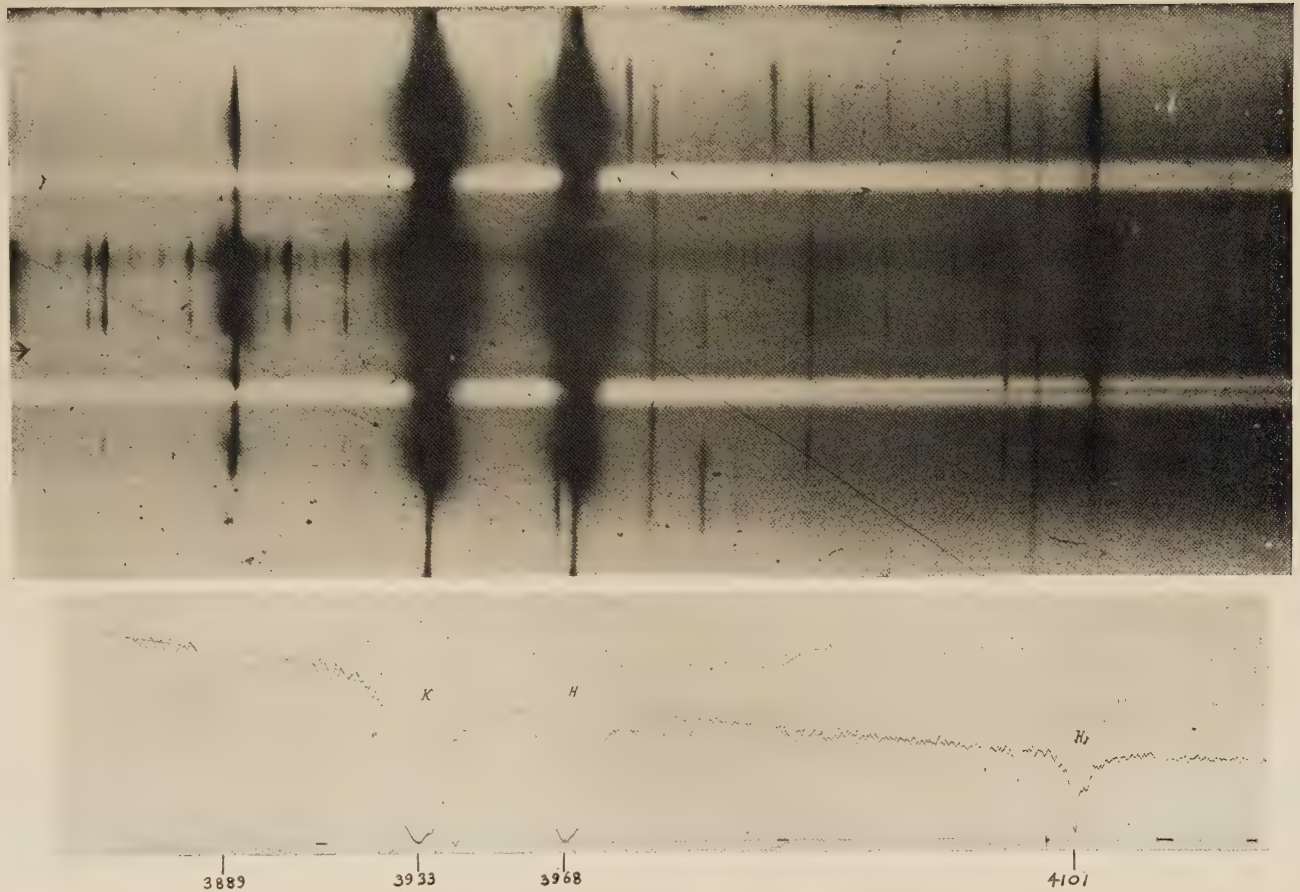
Temperature indicated by Continuous Spectrum.

	$p=0{\cdot}8$.			$p=1{\cdot}0$.	
h.	Plaskett.	Abbot, 1922.	Abbot, 1923.	Plaskett.	Abbot, 1922.
$0{\cdot}025r$	2100° K.	1800° K.	1800° K.	1700° K.	1500° K.
$0{\cdot}031r$	2100	1800	1800	1700	1500
$0{\cdot}036r$	2740	2270	2220	2100	1800
$0{\cdot}044r$	2270	1940	1940	2000	1800

The high ionization revealed by the line spectrum of the prominence shows that the low value of T indicated by this continuous spectrum cannot be a true temperature for the prominence matter. The source of the continuous spectrum is not known, but if it comes from the central part of the prominence the only way in which it could become redder would be by atomic scattering by the outer parts of the prominence. The very definite effect observed, the alteration of the slope of the intensity curve from that of 4000° K or 5000° K to that of 2000° K, is, however, so large that only a very small portion of it, if any, can be attributed to such a cause. The result needs confirmation from a wider range of wave-lengths, and if confirmed the source of the apparent low temperature must be sought for.

It is interesting to note that the temperature indicated by the continuous spectrum increases with the height. This suggests that, whatever the cause of the apparent low temperature in the lowest levels, there comes from the higher levels a larger proportion of sunlight scattered by electrons and corresponding to a black-body distribution for a higher temperature. Ludendorff has shown (*Sitz. D. Preuss. Akad. d. Wiss.*, 1925, 83) that the continuous light of the corona has in general the same distribution as ordinary sunlight. This is to be expected if the coronal light is mainly sunlight scattered by electrons, for Woltjer has shown (*B.A.N.*, **3**, 103, 163, 1926) that the distribution of light so scattered is practically unaffected by the scattering. Our results suggest the presence of a transitional layer between prominences and the upper corona ; in this layer the continuous spectrum changes from that of a body of lower temperature to that of a body of the temperature of the photosphere, as the proportion of light scattered by electrons increases. But this result requires confirmation based on a wider range of spectrum than was available in this case.

6. *Ultra-violet continuous Spectrum due to Hydrogen.*—The distribution of intensity in the continuous spectrum which commences near the head of the Balmer series and extends towards the region of shorter wave-lengths is of considerable interest. Of the various theoretical discussions of this continuous spectrum we select for comparison with observation the theory of Kramers. According to his views the radia-



Continuous spectrum in high prominence or low corona. The arrow on the left shows the point in the spectrum where the photomicrometric trace was taken.

tion of frequency lying between ν and $\nu + d\nu$ from an atom bombarded with cathode rays of velocity v is proportional to $\frac{d\nu}{v}$.* On the hypothesis that the electrons in the chromosphere have a Maxwellian distribution of velocities, the number of emissions in the range $d\nu$ due to electrons of velocity between v and $v + dv$ will be proportional to

$$ve^{-\frac{mv^2}{2kT}}dvd\nu.$$

If ν_0 is the frequency of the limit of the Balmer series and ν' any other frequency under examination the intensity of radiation between frequencies ν' and $\nu' + d\nu'$ will be given by

$$h\nu' d\nu' \int_{v'}^{\infty} ve^{-\frac{mv^2}{2kT}}dv,$$

where $(\nu' - \nu_0)$ is the frequency of the emission corresponding to the velocity v' of the electron. This integral may be written as

$$Ce^{-\frac{mv'^2}{2kT}}d\nu' \quad \text{or} \quad Ce^{-\frac{h(\nu'-\nu_0)}{kT}}d\nu'.$$

In the low flash the continuous spectrum can only be measured from 3646 A to 3480 A, while in a prominence in the flash spectrum it is measurable down to 3343 A and in the prominence on the coronal spectrum to 3323 A. The values of I_λ at the wave-lengths concerned and the resulting temperature are as follows :

				Height.
(*a*) Low flash	I_{3640} 6·7 I_{3480} 4·1	T = 3600° K		400 *km.*
(*b*) Prominence in flash spectrum	I_{3643} 8·1 I_{3343} 2·84	T = 4000° K		700 *km.*
(*c*) Prominence in coronal spectrum	I_{3646} 3·4 I_{3323} 0·93	T = 3200° K		14,000 *km.*

The values given by $p = 1{\cdot}0$ on using Abbot's curves for the solar continuous spectrum are several hundred degrees lower.

A correction has been attempted for the faint continuous spectrum which underlies this hydrogen spectrum in the lower levels. This correction has to be obtained by extrapolating from a curve which is itself not too well determined : the most reliable correction to be made for this appears to be a reduction of the temperature indicated by the hydrogen continuous spectrum by about 200°.

7. *Intensity of the* [K] *Line.*—To one further problem a very tentative answer may be attempted. Schwarzschild in the 1905 eclipse (*Abh. d. k.*

* *Phil. Mag.*, **46**, 836, 1923. Eddington, "The Internal Constitution of the Stars," §§ 156, 160.

Ges. d. Wiss zu Göttingen, **5**, No. 2, 52, 1907) gave as his estimate for the ratio of the total radiation of the [K] line in several prominences to the radiation from one Ångstrom of the Sun's continuous spectrum near [K] figures ranging from 0·1 to 0·5. Owing to the over-exposure of the strong lines in all the lower layers it is quite impossible to check these figures from our eclipse spectra. Only at very considerable heights in the prominence is the [K] line weak enough for its density to be measurable across its width so that the total radiation may be estimated. Even in results for these heights there are two fresh uncertain factors to be taken into account. The Sun's continuous spectrum with which the eclipse spectra have to be compared was photographed with the image-forming mirror stopped down, and the size of the stop adopted in the following work, 2·5 ins., depends upon the observers' memory. The diameter of the mirror was 9 inches, and the factor of 16 has been adopted to allow for the stop. Again the exposure on the Sun was not timed exactly, but it is not likely that an experienced observer would have been very far wrong in estimating a second. As regards the exposure on the flash plates, points far enough along the slit have been chosen to secure that they were exposed to the flash for the whole period of the exposure and not covered by the Moon for part of the time.

Despite the above causes of uncertainty it is thought that it may be of interest to give a measure of the strength of the [K] line. Micrometric traces were secured at two heights in the flash spectrum and measures were made of the density of the line at different points across its breadth. A measure of the total light emitted in the [K] line at each height was then made and compared with the amount of light emitted in one Ångstrom of the solar continuous spectrum at 3950 A. The resulting figures were as follows :

TABLE VI.

Height above Sun's Limb (Flash Spectrum).	$\frac{I_K}{I_{cont.}}$.
$0{\cdot}05r$	$\frac{1}{250}$
$0{\cdot}10r$	$\frac{1}{1940}$

This may be compared with results from the photograph secured for the coronal spectrum. In this latter case (Table VII.) since the slit was not kept fixed relative to the limb of the Sun, any point on the slit at which the density of the [K] line was measured corresponds to a range of heights above the Sun's limb. In this case, owing to the brightening of the spectrum at condensations in the prominence, the higher point gives the stronger radiation.

TABLE VII.

Height above Sun's Limb (Coronal Spectrum).	$\frac{I_K}{I_{cont.}}$.
$0{\cdot}068r - 0{\cdot}036r$	$\frac{1}{4100}$
$0{\cdot}076r - 0{\cdot}053r$	$\frac{1}{1760}$

It should be emphasised that the results in Tables VI. and VII. are very uncertain. They are probably of the right order for the points in the high prominences concerned, and are published at the request of other investigators. The values of the corresponding chromospheric intensities are far too high to measure on our films.

8. *Summary.*—The nature of the valuable results to be obtained from rigidly accurate spectrophotometry applied to eclipse spectra is indicated by this preliminary investigation. Although no great accuracy can be claimed for the present work, its results have not been seriously modified in alternative reductions made with different values for photometric calibration. The chief results found are : (1) a reasonably close agreement between the relative intensities of lines in multiplets in the higher chromospheric spectrum and those given by the multiplet formulæ. (2) A small divergence for the terms of the Balmer series from $l = 6$ to $l = 28$ from Schrödinger and Pauli's formula, the divergence increasing slightly with l and with the depth of the layer of the solar atmosphere examined. The lines fall off in intensity slightly faster than the formula indicates. (3) A temperature of about 800° less than the flash layer for a high prominence, as determined from the relative brightness of lines of different excitation potential in the prominence and in the flash. (4) A temperature of about 4000° for the flash and 3200° for a high prominence as given by applying Kramers' theory to the continuous spectrum of hydrogen, which appears at the head of the Balmer series. (5) For a high prominence or for very low corona a marked deviation from the slope of the intensity curve of the Sun's continuous spectrum over the short range of continuous spectrum 4100 A to 3850 A. The temperature indicated 2000° K is not regarded as a true temperature. (6) Very tentative estimates of the total radiation from the [K] line in a prominence at very considerable heights above the Sun's limb.

All these problems and a wide range of fresh investigations await the attention of future eclipse expeditions. A study of preliminary work for accurate eclipse spectrophotometry has been made at Utrecht and applied in the Dutch expeditions to Sumatra in 1926 (unfortunately spoilt by clouds) and to Lapland in 1927. A discussion of the results so far available has been presented to the Amsterdam Academy by Professor Pannekoek and M. Minnaert and will shortly be published in the *Verhandelingen d. k. Akad. der Wetensch. te Amsterdam* (*English*), 1928. If the requirements of accurate spectrophotometry are fully complied with, many results of interest and value may be expected from future eclipses. It should be added here that the above investigation was made possible by a grant from the Government Grant Committee of the Royal Society towards the expenses of the eclipse expedition to Sumatra.

The Form of an Absorption Line in the Spectrum of a Rotating or Expanding Star. By J. A. Carroll, M.A., Ph.D., Assistant Director of the Cambridge Solar Physics Observatory.

(*Communicated by the Director.*)

1. The spectrum of the light reaching us from a star, obtained by means of the usual astronomical spectrographic apparatus is that of the radiation from the star integrated over its disc. The extreme remoteness of the stars renders any examination of the spectra from different elements of the disc impossible.

The spectrum of the light emitted from an element of the disc in a given direction will not in general be the same as the spectrum of the integrated radiation.

The chief factors causing this difference are (1) the relative Doppler displacement of the spectra from different elements due to rotation or expansion of the star, (2) convection currents in the stellar atmosphere, (3) disturbed areas such as sun-spots and prominences indicate, (4) displacements of the lines of the kind known as "limb effect," and (5) darkening towards the limb.

Of these the first and last are considered in this paper. The effect of disturbed areas cannot well be taken into account, and we must suppose that the lines to which the formulæ developed are applicable are not seriously subject to this irregularity. Our knowledge of the behaviour of lines in the solar spectrum should be of service in choosing suitable quiescent lines. Likewise by choosing lines properly the effects of convection currents may be avoided, and such displacements as are involved in limb effect may be taken to be small so that their effect can either be neglected or investigated separately.

Quite moderate rotational or expansional speeds will produce in a star of average size Doppler displacements between centre and limb of the same order of magnitude as the width of a line.* The effect on the integrated spectrum is to broaden the line and to reduce its central intensity. The effect of darkening towards the limb is to reduce the contribution to the integrated spectrum from the regions near the limb.

Of the two effects of velocity we shall here be concerned with the change in contrast (central intensity) rather than with the increase in line width, which has been studied by Rayleigh and others. The reason for this is that line width is a rather indefinite quantity, not often readily measurable and one not calculable in the present state of the theory of stellar atmospheres.

It will be seen that *a priori* knowledge of relative central intensities yields a possibility of finding the speeds of stellar rotation or expansion, and the true intensity of the line in the normally emergent radiation.

2. We shall consider a symmetrical line whose centre is at a wave-

* See in this connection the work of R. A. Rossiter and D. B. M'Laughlin, who have obtained direct measurements of these displacements in eclipsing binaries. *Ap. J.*, **60**, 15 and 22.

length λ_m. Instead of λ, a more convenient co-ordinate of position in the line is the quantity $\lambda_m - \lambda/\lambda_m$ which we denote by ζ. This quantity is for ordinary lines (of width about one Ångström and in the commonly observed wave-lengths) small compared to unity, being zero at the centre of the line and of order 10^{-4} near the edges.

The intensity I of the light issuing in the direction of the line of sight from an element *ds* of the star may be written as

$$I = s(\theta)I_0(\zeta)\{1 - a(\theta)f(\theta, \zeta)\} \quad . \quad . \quad . \quad (1)$$

where I_0 is the value of I at the centre of the disc, θ is the angle between the line of sight and the normal to the star's surface, s represents the darkening of the continuous background and a the change in contrast of the line over the disc. Since the range in ζ is only over the width of a line we may take I_0 to be constant, and we shall further suppose that the shape of the line ($f(\theta,\zeta)$) is the same at all points of the disc so that f is independent of θ.

The following restrictions on the values of these quantities hold :—

$$\left.\begin{array}{lll} 0<s\leqslant 1 & s(0) = 1 & \\ 0<a<1 & a(0) = a_0<1 & \\ 0\leqslant f(\zeta)\leqslant 1\,; & f(\zeta) = f(-\zeta) & \\ f(0) = 1\,; & f'(0) = 0\,; & f''(0)<0 \end{array}\right\} \quad . \quad . \quad (2)$$

and for the light emitted from the element *ds* we have

$$I = sI_0(1 - af(\zeta)) \quad . \quad . \quad . \quad (3)$$

Let v be the velocity in the line of sight (positive if receding), then light of wave-length λ issuing from *ds* is recorded as of wave-length $\lambda + \beta\lambda$ (where $\beta = v/c$, c = velocity of light), *i.e.* it falls on the photographic plate in a position corresponding to this wave-length.

The light falling on the plate as λ has effectively come from different regions of the absorption line in different parts of the star, that from *ds* behaving as if it were emitted from $\lambda - \beta\lambda$, or $\zeta + \beta(1 - \zeta)$.

Let R be the radius of the star, supposed spherical, and let the observed line be

$$I = 2\pi R^2 I_0 c_0(1 - \mu\phi(\zeta)) \quad . \quad . \quad . \quad (4)$$

ϕ is the observed shape, μ we shall term the observed intensity, and the significance of c_0 will be explained presently.

Then

$$\begin{aligned} 2\pi R^2 I_0 c_0(1 - \mu\phi(\zeta)) &= \int sI_0(1 - af(\zeta + \beta\overline{1 - \zeta}))ds \\ &= 2\pi R^2 I_0 \int_0^1 s\xi d\xi - R^2 I_0 \int saf(\zeta + \beta\overline{1 - \zeta})d\sigma \quad (5) \end{aligned}$$

where ξ is the distance of the element *ds* from the centre of the disc expressed as a fraction of the radius, and $d\sigma$ is an element of this " reduced disc " of radius unity.

$$c_0 = \int_0^1 s\xi d\xi \quad . \quad . \quad . \quad . \quad (6)$$

Thus c_0 is the ratio of the intensity of the continuous background at the centre of the disc to the average intensity.

For the shape and intensity of the line then we have

$$\mu c_0 \phi(\zeta) = \frac{1}{2\pi}\int saf(\zeta + \beta(1 - \zeta))d\sigma \quad . \quad . \quad (7)$$

By a well-known principle the effects of rotation and of expansion can be considered separately if they are small and the results combined by addition. We shall then consider the cases of pure expansion and of pure rotation. Since it will appear that the effects will be small in general this will be justified, when this is not the case no simple expressions can be obtained. The effect of radial velocity is merely to displace the spectrum as a whole so far as we are concerned, since the change in displacement over the width of a single line is negligible.

For a line whose intensity is reduced to $1/e$ of the central intensity in 0·5 Å.U. (half width 0·5 Å.U.) at $\lambda_m = 5000$ Å.U., ζ is 10^{-4} at the half width and so can be taken as $< 10^{-3}$ over the whole range ; velocities of expansion or rotation exceeding 30 km./sec. are scarcely to be expected on various grounds, *e.g.* stability, but allowing up to 300 km./sec. β is less than 10^{-3} so that in (7) we can neglect the term $\beta\zeta$ compared with either β or ζ and reduce it to

$$\mu c_0 \phi(\zeta) = \frac{1}{2\pi}\int saf(\zeta + \beta)d\sigma \quad . \quad . \quad . \quad (8)$$

3. *The Case of a Non-rotating Expanding Star.*—Let V be the outward velocity of the centre of the disc, then the line of sight velocity of ds is $-v\cos\theta$, and the corresponding $\beta = -\frac{V}{c}\cos\theta$, also $\xi = \sin\theta$. Let $\beta_0 = -\frac{V}{c}$ then (8) becomes

$$\mu c_0 \phi(\zeta) = \int_0^{\frac{\pi}{2}} saf(\zeta + \beta_0 \cos\theta)\sin\theta\cos\theta d\theta \quad . \quad (9)$$

$$= f(\zeta)\int_0^{\frac{\pi}{2}} sa\sin\theta\cos\theta d\theta + \beta_0 f'(\zeta)\int_0^{\frac{\pi}{2}} sa\sin\theta\cos^2\theta d\theta$$

$$= c_1 f(\zeta) + c_2\beta_0 f'(\zeta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

neglecting higher powers of β and where c_1 and c_2 represent the values of the integrals.

Thus

$$\mu c_0 \phi(\zeta) = c_1 f\left(\zeta + \frac{c_2}{c_1}\beta_0\right) \quad . \quad . \quad . \quad (11)$$

and the maximum of the observed line will occur at $\zeta = -\beta_0 c_2/c_1$, *i.e.* there will be a resultant Doppler shift of c_2/c_1 times that at the centre of the disc, so that this factor must be applied to the observed displacement to obtain the true velocity of expansion.

For the central intensity we obtain

$$\mu = c_1/c_0 . \quad . \quad . \quad . \quad . \quad (12)$$

the same value as for a non-expanding star to this order.

The velocity of expansion V is then given, if D represents the "Doppler velocity" of the observed spectrum, by

$$V = Dc_1/c_2 \quad . \quad . \quad . \quad . \quad (13)$$

If we neglect darkening and change of contrast ($s = 1, a = a_0$) then $c_1/c_2 = \frac{3}{2}$ or the observed displacement $= \frac{2}{3}$ that due to velocity of expansion.

If we use as a combined darkening law $sa = a_0(2 + 3\cos\theta)/5$ (or $s = (2 + 3\cos\theta)/5$ and a constant, neglecting change of contrast) we obtain $c_1/c_2 = \frac{17}{24}$, the result given by Eddington.*

For a combined law of darkening of the form $\dfrac{a + b\cos\theta}{a + b}$ we find

$$\frac{c_1}{c_2} = \frac{6a+4b}{4a+3b}.$$

Equation (11) also shows that the line is symmetrical about the displaced centre. The net effect then is that the line is displaced but remains unchanged in form or intensity by the expansion.

4. *The Case of a Rotating Non-expanding Star.*—Equation (8) now becomes

$$\mu c_0\phi(\zeta) = \frac{1}{2\pi}\int_0^1\int_{-\pi}^{\pi} saf(\zeta + \beta\xi\sin\phi)\xi d\phi d\xi \quad . \quad . \quad (14)$$

where ξ, ϕ are polar co-ordinates on the reduced disc and $\beta = V\sin\Theta/c$, Θ being the inclination of the star's axis to the line of sight and V its equatorial velocity of spin.

Thus

$$\mu c_0\phi'(\zeta) = \frac{1}{2\pi}\int_0^1\int_{-\pi}^{\pi} saf'(\zeta + \beta\xi\sin\phi)\xi d\phi d\xi$$

and

$$\mu c_0\phi'(0) = \frac{1}{2\pi}\int_0^1\int_{-\pi}^{\pi} saf'(\beta\xi\sin\phi)\xi d\phi d\xi = 0$$

since

$$f'(-\zeta) = -f'(\zeta),$$

i.e. the centre of a symmetrical line is undisplaced.

It is however worth pointing out that the displacement of the centre line is zero to the second order of small quantities for

$$f'(\beta\xi\sin\phi)(1 + \beta\xi d\phi) = \{f'(0) + \beta\xi f''(0) + \ldots\}\{1 + \beta\xi d\phi\},$$

and

$$f'(0) = 0$$

and

$$\int_{-\pi}^{\pi} \beta^2\xi^2 f^4(0)\sin\phi d\phi = 0,$$

* *Internal Constitution of the Stars*, p. 185, Camb. Univ. Press, 1926.

so that terms of $\beta^3\xi^3$ only remain, provided, of course, that the values of $f^4(0)$ and c remain finite.

For the *Central Intensity*

$$\mu c_0\phi(0) = \frac{1}{2\pi}\int_0^1\int_{-\pi}^{\pi} saf(\beta\xi \sin\phi)\xi d\phi d\xi$$

or

$$\mu c_0 = d_1 + \tfrac{1}{4}\beta^2 d_3 f''(0) \quad . \quad . \quad . \quad . \quad (15)$$

where

$$\left.\begin{aligned} d_1 &= \int_0^1 sa\xi d\xi = c_1 \\ d_3 &= \int_0^1 sa\xi^3 d\xi \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (16)$$

since $\phi(0) = 1$ and $f(0) = 1$, $f'(0) = 0$ and we neglect terms in the Maclaurin expansion of f beyond $\beta^3 f'''(0)$, *i.e.*

$$\beta^2 d_3 f''(0) >> \tfrac{1}{16}\beta^4 d_5 f^4(0).$$

In (15) the observable quantity is μ, and we must express $f^4(0)$ in terms of $\phi(\zeta)$, the observed curve. This is readily done as follows :—

$$\mu c_0\phi'' = \frac{1}{2\pi}\int_0^1\int_{-\pi}^{\pi} saf''(\beta\xi d\phi + \zeta)\xi d\phi d\xi,$$

$$\begin{aligned}\mu c_0\phi''(0) &= \frac{1}{2\pi}\int_0^1\int_{-\pi}^{\pi} saf''(\beta\xi \sin\phi)\xi d\phi d\xi \\ &= d_1 f''(0) \quad . \quad . \quad . \quad . \quad . \quad (17)\end{aligned}$$

to the second order.

Thus

$$\mu = \frac{d_1}{c_0} + \frac{1}{4}\beta^2\frac{d_3}{d_1}\phi''(0) \,.\, \mu \quad . \quad . \quad . \quad (18)$$

Now d_1 and d_3 are both proportional to d_0 the depth of the line in the light issuing normally from an element ds. The remaining parts of d_1 and d_3 depend on the laws of darkening and change of contrast. While it is perhaps too sweeping an assumption to take these as constant for all lines, yet we can reasonably suppose that they are the same for all the lines of one close multiplet, since their wave-lengths are nearly the same, and they must all arise in the same places in the star under very closely similar conditions. Writing (18) as

$$\mu = a_0(x) + y \,.\, \beta^2 \,.\, e \,. \quad . \quad . \quad . \quad (19)$$

where e is $\mu\phi''(0)$ we see that a_0 and β^2 are expressed in terms of measured quantities μ and e and theoretically calculable coefficients x and y.

If we have any independent knowledge of a_0 then β is determinable and *vice versa*, the darkening terms x and y being known. By assuming that x and y are the same for all lines of a given multiplet, we can proceed further and find either the true relative intensities, or if these be known, the rotational velocity.

We proceed to work out in detail, to discover the numerical magnitudes involved, the case of a star for which a combined darkening law

$$sa = \frac{2 + 3\cos\theta}{5}$$

is assumed and for a line of the form $e^{-\alpha^2\zeta^2}$.

In this case it is more convenient to use rectangular co-ordinates over the disc so that

$$\begin{aligned}
\mu c_0 \phi(\zeta) &= \frac{1}{2\pi}\int_{-1}^{+1}\int_{-1}^{+1} saf(\zeta+\beta x)dydx \\
&= \frac{1}{2\pi}\int_{-1}^{+1}\int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} + f(\zeta+\beta x)\left\{\frac{2+3\sqrt{1-x^2-y^2}}{5}\right\}dydx \\
&= \frac{1}{2\pi}\int_{-1}^{+1} f(\zeta+\beta x)\left\{\frac{8\sqrt{1-x^2}+3\pi(1-x^2)}{10}\right\}dx \\
&= \frac{1}{20\pi}\Big\{f(\zeta).8\pi + \frac{1}{2!}f''(\zeta).\beta^2.\frac{9\pi}{5} + \frac{1}{4!}f^4(\zeta).\beta^4\frac{59\pi}{70}. \\
&\qquad + \frac{1}{6!}f^6(\zeta).\frac{507\pi}{1008} + \ldots\Big\} \\
&= \tfrac{2}{5}\{f(\zeta) + \beta^2 f''(\zeta)0{\cdot}1125 + \beta^4 f^4(\zeta)0{\cdot}004390 \\
&\qquad + \beta^6 f^6(\zeta)0{\cdot}00008732 + \ldots\}.
\end{aligned}$$

In this case of course $d_1 = 4\pi/5$.

We shall take α as 10^4, so that a line at 5000 Å.U. would have a half width of 0·5 Å.U. (*i.e.* would be reduced to $1/e$ of its central intensity) or in terms of ζ a half width of 10^{-4}.

The figure shows the change produced in a line whose central intensity is taken to be zero (perfect blackness) for no rotation, the changes in any actual case are obtained by multiplying the results obtained by the depth of the absorption line expressed as a fraction of the depth of the continuous background.

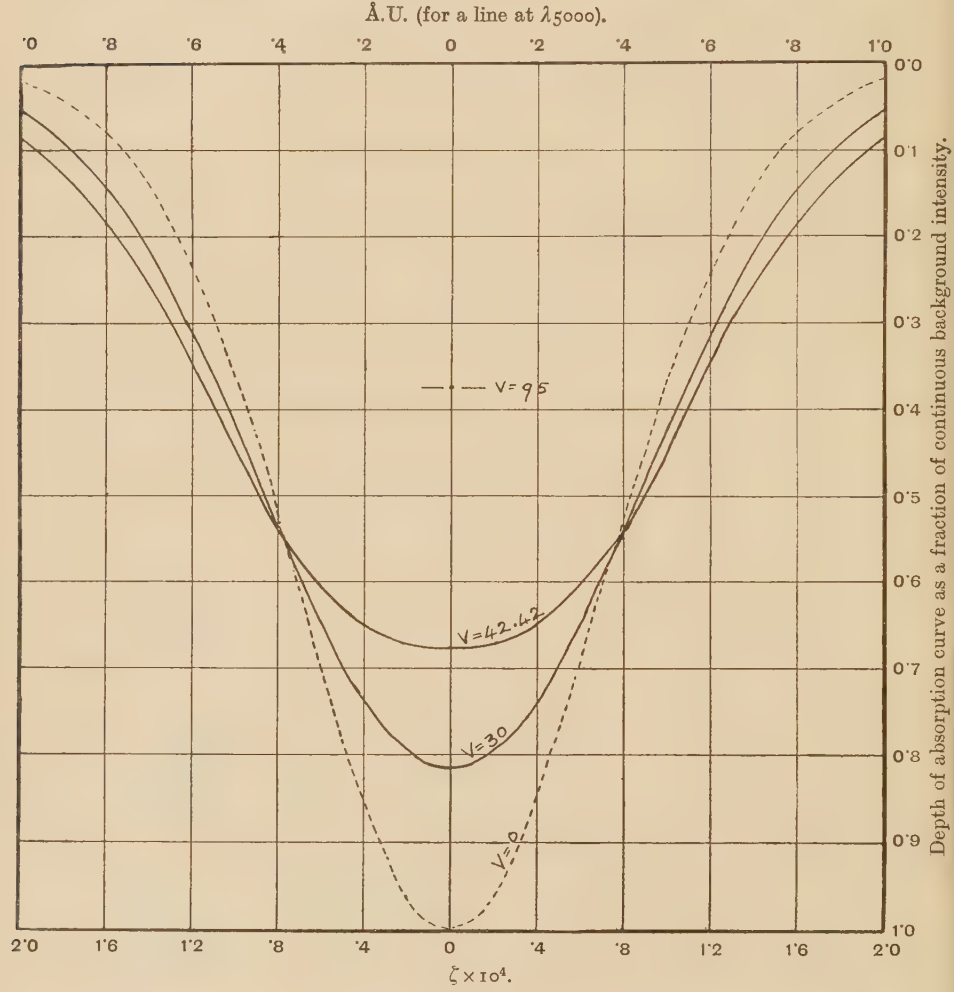
Below are tabulated the changes in central intensity.

Equatorial Velocity. km./sec.	Depth of Centre of Absorption Line of Half Width $\zeta = 10^{-4}$.
0	1·000
30	0·816
42·42	0·677
94·87 *	0·37

It will be seen from the curves, that over the range of application of our approximations the width of the line is not very greatly altered, the principal effect is a filling in and flattening of the centre. The loss of

* Rough extrapolation, the formula ceases to be valid near 65 km./sec.

central intensity increases slowly at first, then rapidly, and since our results apply to any line of half width $\zeta = 10^{-4}$ we can make one deduction from observations in the light of these results. Lines of half


Å.U. (for a line at λ5000).
·0 ·8 ·6 ·4 ·2 0 ·2 ·4 ·6 ·8 1·0
0·0 0·1 0·2 0·3 0·4 0·5 0·6 0·7 0·8 0·9 1·0
Depth of absorption curve as a fraction of continuous background intensity.
V=95
V=42.42
V=30
V=0
2·0 1·6 1·2 ·8 ·4 0 ·4 ·8 1·2 1·6 2·0
ζ × 10⁴.

The form of an absorption line of half width 0·5 Å.U. at λ5000 for various equatorial velocities of rotation V.

width 0·5 Å.U. are common in stellar spectra, and while the resolving powers employed are usually adequate for such lines, yet the filling up of the line from this and other instrumental causes is usually quite appreciable, yet without allowing for this the spectro-photometer tracings so far published show that lines have a central intensity in general

(greater) darker than half the continuous background. I have not found any spectrum in which every line was weaker than this, and we are thus able to say that no stars have been found rotating with an equatorial velocity much greater than say 50 km./sec., and that since these are usually lines of intensity 0·9 the rotational speed must have been more nearly of the order of 10 km./sec. or less. Data for more precise estimates are lacking, and since the speeds to be expected are then small the application of the method must await more refined photometric measurements.

The figures for the central intensity may be applied to other lines and velocities providing the product $\alpha\beta$ is constant, *e.g.* instead of 30 km./sec. and a line of half width 0·5 Å.U. we can choose 3 km./sec. and a line 0·05 Å.U. wide for which the reduction in intensity is the same.

5. *Note on Expansion used to obtain* (15). For a symmetrical line the odd differential coefficients vanish, and in any event if they be not infinite the terms involving them vanish leaving as the general term of the series for $\phi(0)$,

$$\frac{1}{2\pi}\int_0^1\int_{-\pi}^{\pi} saf^{2n}(0)\,\frac{1}{(2n)!}\,\beta^{2n}\sin^{2n}\theta\xi^{2n+1}d\theta d\xi$$

$$= \frac{\beta^{2n}f^{2n}(0)}{2^{2n}(n!)^2}\int_0^1 sa\xi^{2n+1}d\xi$$

$$< \frac{\beta^{2n}f^{2n}(0)}{2^{2n}(n!)^2}\cdot\frac{1}{2n+2}.$$

Thus a sufficient condition for convergence is

$$\beta^{2n}f^{2n}(0)/2^{2n+1}(n+1)!\,n! \leqslant 1$$

for then each term is $<1/n(n+1)$.

If the line is symmetrical $|f^{2n}(\zeta)| \leqslant |f^{2n}(0)|$ and so R the remainder is $<\beta^{2n}f^{2n}(0)/2(n+1)!\,n!$

Summary.

Expressions are obtained connecting the observed form of stellar absorption lines with the form emitted normally from an element of the disc.

Methods of determining from spectro-photometric observations the intrinsic depth of a line and the equatorial velocity of stellar rotation or expansion are developed.

It is shown that knowledge of the relative intensities of a set of lines, such as a multiplet, would yield a method of determining the rotational speed in the absence of *a priori* knowledge of the intrinsic intensity of a single line.

Available measurements support the view that stellar rotational speeds are low.

Large Magnetic Storms and Large Sunspots. By W. M. H. Greaves, M.A., and H. W. Newton.

(*Communicated by the Astronomer Royal.*)

This paper gives details of a comparison between large magnetic storms and large sunspots during the period 1874 April to 1927 December, and is to some extent an extension of a paper by E. W. Maunder dealing with the period 1875 to 1903 in *Monthly Notices*, **64**, 1904, 205, 222. Systematic solar observations were started at Greenwich in 1874 April, and consequently there are now available for discussion over fifty years of solar and magnetic records.

1. In order to carry out a comparison, two catalogues were prepared independently; one consisting of all spot groups of mean area 500 millionths of the sun's hemisphere or greater, the other containing all magnetic disturbances with ranges equal to or greater than 30′ for Declination or 150γ for Horizontal Force (or North Force for the years 1916 to 1925) or Vertical Force. A spot of area not less than 500 millionths is generally visible to the naked eye when near the centre of the disc, and it is convenient to refer to such groups as are discussed here as "naked-eye spots." A few groups, whose mean area was rather less than 500 millionths but whose maximum area was considerably more, were included in the catalogue of spots. The total number of spots catalogued was 455, and the total number of magnetic storms 405.

The present communication represents an attempt to investigate to what extent individual storms bear a relation to individual spots. For this purpose it seems of little avail to discuss the smaller storms, and accordingly a restricted list of storms was prepared, consisting of those whose ranges were equal to or greater than one degree in declination *or* 300γ in either of the other elements. In a few cases in which a storm exceeded the prescribed limit by a small amount in one element only, and that by virtue of a single swing, the storm was not included in the restricted list. This list consists of 61 storms, and it is these which are discussed here. The remaining 344 smaller storms are available for statistical treatment, and it is hoped to discuss them in a subsequent paper. It may be mentioned, however, that their distribution roughly follows the solar cycle, and this circumstance in itself would seem to be sufficient to indicate a solar origin for magnetic disturbances.

2. Table I. gives the 61 storms in question. The first column gives the dates of the more intense portions of the storms; the second gives the time of commencement. The majority of these storms begin with the characteristic movement known as "sudden commencement" or "S.C." In seven cases where a "sudden commencement" was not identifiable the times of commencement are given in brackets. The third column gives the ranges for the separate magnetic elements, the sign > being prefixed in cases where the whole range could not be

TABLE I.

No.	Date.	STORM. G.M.T. of Commencement (0h = Midnight).	STORM. Ranges of D, H (or N), and V.	SUNSPOTS. Greenwich Number.	Mean Area.	Central Meridian Passage.	Distance from Centre of Disc at C.M.P.
		d h				d	°
1	1874 Oct. 3–4	Oct. 3 13·9	57′ : > 235γ : 325γ	127	984	Oct. 4·5	20 S.
2	1880 Aug. 12	Aug. 11 10·3	66′ : > 300γ : 385γ				
3	1881 Jan. 31	(Jan. 30 23)	75′ : 330γ : 365γ				
4	1881 Sept. 12–13	(Sept. 12 13)	65′ : 335γ : 345γ	566	519	Sept. 10·0	7 N.
5	1882 Apr. 16–17	Apr. 16 23·6	> 59′ : > 455γ : > 1040γ	728	508	Apr. 13·2	13 S.
				726	833	Apr. 17·0	14 S.
				729	1744	Apr. 19·1	23 S.
6	1882 Apr. 20	Apr. 20 3·7	> 68′ : > 300γ : > 340γ	726	833	Apr. 17·0	14 S.
				729	1744	Apr. 19·1	23 S.
7	1882 June 24	June 24 12·2	50′ : 325γ : 235γ				
8	1882 Aug. 4–5	Aug. 4 15·8	36′ : 335γ : 260γ				
9	1882 Nov. 17–18	Nov. 17 10·2	115′ : > 565γ : > 1060γ	885	1711	Nov. 18·9	17 N.
10	1882 Nov. 20	(Nov. 20 0·3)	85′ : > 1090γ : > 1040γ				
11	1883 Apr. 3–5	Apr. 3 9·0	58′ : 190γ : 315γ	987	536	Apr. 3·3	20 N.
12	1883 Sept. 16–17	Sept. 16 2·7	52′ : 345γ : > 240γ	1114	920	Sept. 13·9	29 S.
13	1884 July 2–4	July 2 19·3	41′ : 330γ : 295γ				
14	1885 Mar. 15–16	Mar. 15 10·7	54′ : 170γ : 370γ				
15	1885 May 25–26	May 25 13·6	31′ : 360γ : 125γ	1672	516	May 24·3	15 N.
16	1886 Mar. 30–31	Mar. 30 8·4	73′ : > 335γ : 285γ	1860	487*	Mar. 30·8	10 S.
17	1891 May 13–16	May 13 8·3	52′ : 340γ : > 330γ				
18	1892 Feb. 13–14	Feb. 13 5·5	> 73′ : > 540γ : > 635γ	2421	2402	Feb. 11·9	22 S.
19	1892 Mar. 6–7	Mar. 6 9·8	49′ : > 290γ : 325γ				
20	1892 Mar. 11–13	Mar. 11 22·6	74′ : 295γ : 375γ				
21	1892 Apr. 25–27	(Apr. 25 15½)	59′ : 390γ : 395γ	2486	560	Apr. 23·9	15 N.
22	1892 May 18–19	May 18 8·4	75′ : 300γ : 480γ	2515	742	May 16·7	12 S.
23	1892 June 27	June 27 5·0	36′ : 305γ : > 315γ				
24	1892 July 12–14	July 12 17·8	35′ : 520γ : 285γ	2581	1591	July 10·2	7 N.
				2583	829	July 14·1	35 S.
25	1892 July 16–17	July 16 12·7	89′ : 590γ : 370γ	2583	829	July 14·1	35 S.
26	1892 Aug. 12–13	Aug. 12 12·7	75′ : 540γ : 405γ				
27	1893 Aug. 18	Aug. 18 0·2	34′ : 250γ : > 345γ				
28	1894 Feb. 23–24	Feb. 22 22·3	52′ : > 260γ : 390γ	3412	1208	Feb. 22·5	25 S.
29	1894 Feb. 25–26	Feb. 25 7·8	74′ : 400γ : > 270γ	3412	1208	Feb. 22·5	25 S.
30	1894 Feb. 28–Mar. 1	Feb. 28 15·2	49′ : 340γ : > 230γ				

* Maximum area 849 on Apr. 2.

TABLE I.—*continued.*

No.	Date.	STORM. G.M.T. of Commencement (0h = Midnight).	STORM. Ranges of D, H (or N), and V.	SUNSPOTS. Greenwich Number.	Mean Area.	Central Meridian Passage.	SUNSPOTS. Distance from Centre of Disc at C.M.P.
		d h				d	°
31	1894 Mar. 30–31	(Mar. 30 17)	57′ : 325γ : 470γ	3464	926	Apr. 2·5	28 N.
32	1894 July 20–21	July 20 6·1	63′ : 690γ : > 520γ	3624	553	July 17·2	17 S.
33	1894 Aug. 20	Aug. 20 2·9	66′ : > 445γ : > 530γ	3668	1246	Aug. 17·2	0
34	1894 Sept. 14–15	Sept. 14 1·1	40′ : 300γ : 430γ				
35	1894 Nov. 13–14	Nov. 13 14·1	50′ : 350γ : 500γ				
36	1898 Mar. 15–16	Mar. 15 0·8	83′ : > 345γ : > 520γ	4702	970	Mar. 11·5	6 S.
37	1898 Sept. 9	Sept. 9 12·1	57′ : > 365γ : 480γ	4781	1556	Sept. 9·2	19 S.
38	1900 May 5	May 5 3·2	31′ : 210γ : 370γ				
39	1903 Oct. 31–Nov. 1	Oct. 31 6·0	119′ : 1175γ : 1440γ	5098	466*	Oct. 31·8	24 S.
40	1903 Dec. 13–14	Dec. 13 12·4	60′ : > 135γ : > 355γ				
41	1907 Feb. 9–10	Feb. 9 14·3	95′ : > 225γ : > 460γ	6104	512	Feb. 7·3	8 S.
				6103	650	Feb. 10·0	7 S.
				6108	1619	Feb. 12·4	10 S.
42	1908 Sept. 11–12	Sept. 11 21·7	84′ : > 355γ : > 445γ	6520, 6522†	701	Sept. 10·0	6 N.
43	1908 Sept. 29–30	Sept. 29 13·6	65′ : 400γ : 380γ				
44	1909 Jan. 3–4	Jan. 3 11·5	71′ : > 120γ : 150γ	6591	641	Jan. 3·2	12 N.
45	1909 May 14–15	May 14 4·8	50′ : 430γ : > 355γ	6669	681	May 13·8	13 S.
46	1909 Sept. 25	Sept. 25 11·6	193′ : 1710γ : > 1080γ	6728	605	Sept. 23·8	12 S.
47	1915 June 17–18	June 17 1·8	71′ : > 375γ : > 295γ	7304	767	June 21·0	16 N.
48	1917 Aug. 9–10	Aug. 9 4·3	55′ : 315γ : 225γ	8181	2389	Aug. 10·1	9 N.
49	1917 Aug. 13–15	Aug. 13 14·4	41′ : 380γ : 335γ	8181	2389	Aug. 10·1	9 N.
50	1918 Aug. 15–16	Aug. 15 15·8	30′ : 370γ : 155γ	8631	933	Aug. 16·7	3 N.
51	1919 Aug. 11–12	Aug. 11 6·9	58′ : > 360γ : > 720γ	8994	543	Aug. 9·2	13 S.
52	1920 Mar. 4–5	Mar. 4 11·6	64′ : 240γ : 280γ				
53	1920 Mar. 22–23	Mar. 22 9·2	86′ : 730γ : > 490γ	9143	1966	Mar. 21·6	2 N.
54	1921 May 13–17	May 13 13·2	> 100′ : > 740γ : > 460γ	9334	1324	May 14·7	3 N.
55	1926 Jan. 26–27	Jan. 26 16·4	62′ : > 420γ : > 205γ	9861	3285	Jan. 24·5	26 N.
56	1926 Feb. 23–25	(Feb. 23 16·4)	60′ : 305γ : > 205γ				
57	1926 Apr. 14–15	Apr. 14 14·2	60′ : 455γ : > 205γ				
58	1926 Oct. 15–16	Oct. 13 19·4	111′ : > 430γ : > 580γ	10062	986	Oct. 11·1	12 N.
				10065	527	Oct. 13·7	11 N.
59	1927 July 21–22	July 21 21·1	60′ : 315γ : 335γ				
60	1927 Aug. 19–21	(Aug. 19 12)	> 73′ : 170γ : 205γ				
61	1927 Oct. 12	Oct. 12 10·4	55′ : > 350γ : 230γ				

* Maximum area 807 on Oct. 28.

† From the photographs it is evident that these two groups are an entity.

measured owing to loss of trace. The fourth gives the Greenwich numbers of any "naked-eye" groups of spots which crossed the sun's central meridian within 4 days of the commencement of the storm. The mean area and time of central meridian passage of any such spots are tabulated in the next two columns, whilst the last column gives their distance from the centre of the disc at the time of meridian passage. All particulars relating to spots are abstracted from the *Greenwich Photo-Heliographic Results*.

Counting Nos. 9 and 10,* 1882 November 17–20, as one storm, there are 60 storms, and of these 36 begin within 4 days of the central meridian passage of a "naked-eye" spot. Consequently, we cannot claim so far to have established a universal connection between large storms and large spots. At the same time, it seems as if there is more than a chance connection between the phenomena.

The storms tend to occur near sunspot maximum, and at such a period if 8 days (*i.e.* 4 days on each side of the C.M.) are selected at random, there is a certain probability that a naked-eye group will transit the central meridian during that time. In point of fact, during the 26 individual years in which these 60 great storms are found, there were 331 naked-eye spot groups or 1 spot group in 29 days on the average; therefore, even if there were no connection between storms and spots there would be an expectation of $8/29 \times 60$, or, say, 17 occasions† on which a storm would commence within 4 days of the C.M. passage of a spot. Actually, 36 such coincidences have been found. The numbers are rather too small to be conclusive, but there seems to be a strong suspicion of a connection.

3. *Very Large Storms.*—It is natural to enquire whether a more definite connection can be found by limiting the storms still further. Table II. is extracted from Table I., and is simply a list of those storms which are known to have ranges of 90′ or more in declination or 500γ or more in the other elements, together with "naked-eye" spots transiting the C.M. within 4 days of the commencement of the storm.

This list of very great storms is not necessarily complete, as some storms of great range may not have been included owing to loss of trace. It consists of all storms *known* to have exceeded the above limits.

There are 17 storms in Table II., and 15 are in conjunction with "naked-eye" spots. The exceptions are No. 26, 1892 August 12–13, and No. 35, 1894 November 13–14.

In the case of the storm of 1892 August 12–13 a spot of mean area 277 crossed the central meridian on August 11·5. When observed near the limbs, however, this spot was seen to be associated with a large area of faculæ, and this in itself suggests that the spot had previously been a large one. Actually, the spot in question, No. 2615, is a return of 2583, which was a large complex group of area 829 millionths

* On the traces there is continuous disturbance from Nov. $17^d\ 10^h$ to Nov. $21^d\ 3^h$ with two pronounced maxima at Nov. $17^d\ 10^h$ to Nov. $18^d\ 12^h$ and Nov. $20^d\ 0^h$ to Nov. $20^d\ 21^h$.

† More strictly, the mathematical expectation is rather less, owing to the possibility of more than one spot occurring in conjunction with the same storm.

and associated with the magnetic storm of 1892 July 16–17, itself one of these very large storms. Measures of faculæ associated with spot No. 2615 are—East limb, August 5, 6, 7, area 850; West limb, August 16, 17, area 1100.

TABLE II.

STORM.		SPOT.	
No.	Date.	Greenwich Number.	C.M.P.
5	1882 Apr. 16–17	728	Apr. 13·2
		726	Apr. 17·0
		729	Apr. 19·1
9 and 10	1882 Nov. 17–20	885	Nov. 18·9
18	1892 Feb. 13–14	2421	Feb. 11·9
24	1892 July 12–14	2581	July 10·2
		2583	July 14·1
25	1892 July 16–17	2583	July 14·1
26	1892 Aug. 12–13		
32	1894 July 20–21	3624	July 17·2
33	1894 Aug. 20	3668	Aug. 17·2
35	1894 Nov. 13–14		
36	1898 Mar. 15–16	4702	Mar. 11·5
39	1903 Oct. 31–Nov. 1	5098	Oct. 31·8
41	1907 Feb. 9–10	6104	Feb. 7·3
		6103	Feb. 10·0
		6108	Feb. 12·4
46	1909 Sept. 25	6728	Sept. 23·8
51	1919 Aug. 11–12	8994	Aug. 9·2
53	1920 Mar. 22–23	9143	Mar. 21·6
54	1921 May 13–17	9334	May 14·7
58	1926 Oct. 15–16	10062	Oct. 11·1
		10065	Oct. 13·7

The storm of 1894 November 13–14 is, on the other hand, so far as the evidence of the Greenwich photographs goes, not accompanied at the time by any striking solar disturbance. The spot groups on the disc on November 13 are as follows :—

No.	Mean Area.	C.M.P.
3786	135	Nov. 9·8
3798	8	[Nov. 10]
3793	147	Nov. 10·5
3792	36	Nov. 13·8
3795	63	Nov. 14·2
3797	34	Nov. 17·9

All these spots are quite small; the areas given are mean areas and not the areas for November 13, the latter being as follows:—29, 7, 238, 19, 117, 59, giving a total area for the day of 469. The mean daily area for the year 1894 is 1282. Faculæ of area about 350 millionths, observed at the limbs on November 7 and 16 at latitude 7° S., crossed the central meridian on November 12, but there is nothing unusual in faculæ of this size occurring at this epoch of the solar cycle.

Here is undoubtedly an example of a very large magnetic storm, with a range of 500γ in vertical force, occurring at a time when there was no unusual activity visible on the sun's disc, the activity being, indeed, far below the average for the year. It may be remarked that this storm was not included by Maunder in his list of "Great Storms" given in the paper already referred to, as he confined himself to storms for which the range in declination exceeded a degree (two cases in which the D range was under 1° but exceeded 55′ and the H range exceeded 500γ were included by him). The ranges for the present storm are D 50′, H 350γ, V 500γ.

On reference to the catalogue of 405 storms which has been compiled, it is found that two smaller disturbances commenced 28 days and 56 days respectively before this storm of November 13–14. These disturbances are catalogued as 1894 October 16–20 and September 18–23, but as their ranges are smaller than the limit adopted for the storms discussed in this paper they do not appear in Table I. It may be mentioned, however, that in neither instance was there any conspicuous solar disturbance which could be associated with them. The facts seem to suggest an area of the sun which persisted through three solar rotations, producing a storm on each occasion, but which did not manifest itself as a large spot at the time. It may be noted, however, that a large spot appeared subsequently, viz. Group 3827, mean area 705, C.M.P. 1894 December 10·3, *i.e.* 27 days after the large storm. This spot was not a return, but originated between December 9 and 10.

The circumstance that 16 out of the 17 very large magnetic storms occur within 4 days of the central meridian passage of a striking solar disturbance, points to an individual connection between magnetic storms and spots. But the 17th case is a reminder that a large storm may originate from a solar disturbance which manifests itself as a small spot only, or possibly from something which does not manifest itself as a spot at all.

4. Returning to the more general list of 60 large storms, we have found that 36 commenced within 4 days of the central meridian passage of a "naked-eye" spot group. Of the remaining 24 storms, 7 occurred within 4 days of the C.M.P. of a limited region of the sun, which was characterised, at one or both of the sun's limbs, by patches of faculæ of 1000 millionths of the hemisphere or more in area. The case of the storm of 1892 August 12–13 has already been mentioned. The particulars of the other five cases are as follows:—

(a) *Storm of* 1885 *March* 15.—Spots Nos. 1622 and 1624 of mean area 232 and 124 crossed the C.M. on March 16·2 and March 17·2 respectively. On March 21 and 22 these spots were near the west limb

and were surrounded by a patch of faculæ, area 1200 ; the area near the east limb on March 11 and 12 was measured as 800.

(b) *Storm of* 1892 *March* 11–13.—Spot No. 2440, mean area 386, C.M.P. March 10·6, was seen surrounded by faculæ on March 5–6 near the east limb and on March 15–16 near the west limb. The area of faculæ was 2000 in each case. The spot was the return of a large spot, No. 2421, mean area 2402, C.M.P. February 11·9, which is associated with the magnetic storm of 1892 February 13–14.

(c) *Storm of* 1892 *June* 27.—Spots Nos. 2565 and 2567, mean areas 212, 337, C.M.P. June 23·8, June 25·0, were surrounded by a patch of faculæ on June 19 and 20 near the east limb and June 28–30 near the west limb ; areas, 1200 and 1000 respectively. In addition, a large area of faculæ without a spot (though evidently connected with a large spot, No. 2534, of the previous rotation) was seen in latitude 19° S. on June 20–21 at the east limb and on June 30 and July 1 at the west limb ; areas, 1100 and 950 respectively. The area crossed the central meridian on June 26.

(d) *Storm of* 1908 *September* 29–30.—Spot No. 6533, mean area 399, C.M.P. September 26·8, was accompanied by a large area of faculæ on September 21, 22, and 23 near the east limb (area 2000) and on October 1 and 2 near the west limb (area 1800). Group 6533 is the return of 6510, mean area 1420, C.M.P. August 31·2.

(e) *Storm of* 1926 *February* 23–25.—Spot No. 9881, mean area 390, C.M.P. February 21·3, was surrounded by a large area of faculæ * on February 16–17 at the east limb (area 2000) and on February 25–27 at the west limb (area 2100). Group 9881 is a return of 9861, whose mean area is 3285 and C.M.P. is January 24·5. Group 9861 was associated with the storm of January 26–27.

(f) *Storm of* 1927 *October* 12.—Two spots in latitudes 11° S. and 18° S., mean areas 360 and 200 respectively, C.M.P. October 10·7 and October 11·5, were surrounded by a large patch of faculæ on October 6 near the east limb (area 1000) and on October 15 at the west limb (area 1900). These spots are the returns of large spots, mean areas 810 and 835, and C.M.P. September 13·9 and September 14·4 respectively.

It may be remarked that large areas of faculæ have never been seen to originate on the visible hemisphere except in conjunction with spots, so that when a large area of faculæ is observed with no spot we can infer the previous existence of a large spot, which in some instances may have originated and died out, or reached its maximum, on the invisible hemisphere. If the existence of faculæ be admitted as a criterion of solar disturbance, the number of cases in which a storm occurs in conjunction with the transit of a pronounced disturbed area across the sun's central meridian has now been increased to 43 out of 60.

Of the remaining 17 storms, one, viz. 1894 September 14–15, occurred

* Royds gives details, in *Monthly Notices,* **86**, 1926, 380, of an unusual outburst of bright filaments in this region on 1926 February 22, depicted on spectroheliograms taken at Kodaikanal.

in association with a spot group which had been large in the previous rotation, but whose area of surviving faculæ was less than 1000 millionths. The group in question is Nos. 3703–04, mean area 139, C.M.P. September 12·7, and is the return of No. 3668, mean area 1246, C.M.P. August 17·2, which was associated with the storm of August 20.

In seven other cases the storm was followed one solar rotation later by the transit of a " naked-eye " spot. These instances are as follows :—

STORM.	SPOT GROUP 1 SOLAR ROTATION LATER.		
	No.	Area.	C.M.P.
1880 Aug. 12	355	615	Sept. 9·8
	357	699	Sept. 11·2
1882 Aug. 4–5	830	577	Sept. 2·2
1893 Aug. 18	3173	589	Sept. 13·0
1894 Nov. 13–14 *	3827	705	Dec. 10·3
1926 Apr. 14–15	9943	690	May 13·7
1927 July 21–22	..	790	Aug. 14·2
1927 Aug. 19–21	..	810	Sept. 13·9
		835	Sept. 14·4

Group 355 was a return of 344, area 113, C.M.P. August 14·2 ; the other eight groups are new, but may conceivably indicate the presence of a disturbed area in the previous rotation.

The remaining 9 storms all occur in conjunction with some kind of solar disturbance, as manifested by small spots or moderate areas of faculæ, such as are found on the sun on nearly any day other than in or near minimum years.

5. *Time Interval between Spot and Storm.*—The cases have been selected in which there is *only one* " naked-eye " spot group which crossed the central meridian within 4 days of the commencement of the storm. These occasions have been tabulated below, together with the interval in days between the transit of the spot across the central meridian and the commencement of the storm ; this interval is taken as positive when the spot transits before the storm begins.

These 30 cases are hardly enough to give a statistical result, but, so far as they go, they indicate a tendency of the storm to commence about a day following the transit of the spot across the sun's central meridian. In Maunder's discussion of 19 storms, 1875–1903, a mean interval of over one day was obtained. It is possible that a more reliable result will be obtained from the discussion of the 344 smaller storms which have been catalogued. It may be mentioned that 12 of the above storms are in Maunder's list, so that the two results are not independent. If these 12 storms are excluded from the analysis, the mean interval would be approximately half a day.

* *See* reference to this storm in paragraph 3.

TABLE III.

Storm.	Interval. d	Storm.	Interval. d
1874 Oct. 3–4	−0·9	1898 Mar. 15–16	+3·5
1881 Sept. 12–13	+2·5	1898 Sept. 9	+0·3
1883 Apr. 3–5	+0·1	1903 Oct. 31–Nov. 1	−0·5
1883 Sept. 16–17	+2·2	1908 Sept. 11–12	+1·9
1885 May 25–26	+1·3	1909 Jan. 3–4	+0·3
1886 Mar. 30–31	−0·5	1909 May 14–15	+0·4
1892 Feb. 13–14	+1·3	1909 Sept. 25	+1·7
1892 Apr. 25–27	+1·7	1915 June 17–18	−3·9
1892 May 18–19	+1·7	1917 Aug. 9–10	−0·9
1892 July 16–17	+2·4	1917 Aug. 13–15	+3·5
1894 Feb. 23–24	+0·4	1918 Aug. 15–16	−1·0
1894 Feb. 25–26	+2·8	1919 Aug. 11–12	+2·1
1894 Mar. 30–31	−2·8	1920 Mar. 22–23	+0·8
1894 July 20–21	+3·1	1921 May 13–17	−1·1
1894 Aug. 20	+2·9	1926 Jan. 26–27	+2·2
		Mean	+0·9

6. *Recurrence of Storms.*—With the object of investigating any tendency which may exist for a magnetic storm to be followed by another disturbance approximately one synodic solar rotation period (about 27 days) later, the ranges have been measured of the declination traces for the 25th, 26th, 27th, 28th, and 29th traces * after the trace on which each storm commenced. Cases in which a storm in our catalogue commenced on any one of these traces have been excluded from the measurements. There were 10 such cases, the details being as follows (when the 2nd storm is one of the 60 discussed in this paper, the letter G (for "great") is attached):—

TABLE IV.

1st Storm.	2nd Storm.	Interval between Commencements. d.
1892 Feb. 13–14	1892 Mar. 11–13 G	27·7
1892 June 27	1892 July 26–29	28·8
1892 July 16–17	1892 Aug. 12–13 G	27·0
1894 Aug. 20	1894 Sept. 14–15 G	24·9
1907 Feb. 9–10	1907 Mar. 10–11	28·6
1909 Jan. 3–4	1909 Jan. 29–Feb. 1	26·0
1909 Sept. 25	1909 Oct. 22–23	27·5
1917 Aug. 9–10	1917 Sept. 5	27·1
1926 Jan. 26–27	1926 Feb. 23–25 G	28·0
1927 July 21–22	1927 Aug. 19–21 G	28·6

* A trace normally extends from noon to noon approximately.

In addition, there was one instance (the storm of 1920 March 22–23) in which a disturbance commenced before the 25th trace following the commencement and persisted into the interval being investigated. In this case also the ranges were not measured. Excluding these cases, the mean values of the declination ranges for the successive traces given by 47 * storms are as follows :—

No. . .	25	26	27	28	29
Range . .	12′·8	13′·4	13′·2	12′·2	12′·3

The mean daily range in declination for the 26 years during which our 60 large storms occurred is 14′—taken over all years it is slightly less, about $13\frac{3}{4}'$.

The figures show a very slight tendency to rise to a maximum at the 26th and 27th trace, but in themselves could hardly be regarded as proving anything. They are, however, confirmatory of the results of Chree and Stagg,† who from an analysis of the de Bilt character figures, 1906–25, have found a slight but decided tendency for magnetic disturbance to recur at an interval of 27 days.

Out of the 60 large storms there are 10 cases, already enumerated in Table IV., in which the storm is followed by another storm commencing between the 25th and 29th traces following that on which the original storm commenced. Now, in the 26 years in which these 60 storms occurred there are listed in our catalogue 268 storms in all, or one storm in 35 days on the average. Clearly, chance could easily account for the 10 cases in which a 2nd storm occurred within the period covered by the 25th to 29th trace after one of the large storms. On the other hand, the fact that in 5 cases the 2nd storm is itself one of the 60 large storms is surprising if chance be the only factor, and, also, it should be noticed that 6 out of the 10 intervals in Table IV. lie between $26^{d}\cdot0$ and $28^{d}\cdot0$.

The evidence is unsatisfactory, but on the whole there seems to be a small tendency for these large storms to be followed by a further disturbance after one synodic solar rotation period. But Maunder found a distinct tendency for a magnetic disturbance to be followed by another about 27 days later, and has drawn attention to a number of instances in which magnetic disturbance is associated with a certain meridian of the sun through several rotations.‡ This conclusion was reached by Maunder from a discussion of 276 magnetic disturbances occurring between 1882 and 1903. Most of the disturbances are quite small, and it is therefore possible that a discussion of the smaller storms in our catalogue will yield more definite results as regards tendency to recurrence. In fact, a preliminary examination of the catalogue shows the presence of several sequences of storms of the type to which Maunder directed attention. For the present we have to record the fact that the tendency of the larger storms to be followed

* For two storms the ranges could not be measured owing to loss of the D trace.

† *Phil. Trans.*, **227**, A, 1927, 21–62.

‡ *Monthly Notices*, **65**, 1904, 19–22 ; **76**, 1915, 68.

27 days later by a disturbance striking enough to be called a " storm " is, if it exists at all, slight.

7. *Very Large Sunspots.*—We have been comparing a list of 60 magnetic storms with a list of 455 " naked-eye " spots. If we restrict ourselves to spot groups with mean area greater than 1000 millionths of the hemisphere, we get a list of 65 spots, which is a number comparable with that of large storms. But working with this list of 60 large storms and 65 very large spots, the number of cases in which one of the storms commences within 4 days of the central meridian passage of one of the spots is reduced to 15, a number, however, which is greater than the expectation given by pure chance. These 15 cases are (see Table I.) Nos. 5, 6, 9 and 10, 18, 24, 28, 29, 33, 37, 41, 48, 49, 53, 54, 55. In three instances, viz. Nos. 5 and 6, 28 and 29, 48 and 49, the same large spot occurred in proximity to two of the storms. Thus, out of the 65 very large spots 12 are in proximity to large storms, and of the remaining 53, 15 transit the central meridian within 4 days of the commencement of one of the 344 smaller storms which have been catalogued.

Table V. gives details relating to these spots of mean area greater than 1500. The successive columns give the Greenwich number of the spot group, the year of occurrence, the mean area in millionths of the sun's hemisphere, the time of central meridian passage, and any magnetic disturbance in our catalogue which commenced within 4 days of the C.M.P. The letter G denotes that the storm is one of the 60 large storms which form the theme of the present paper. It will be noticed that in 7 cases out of 23 there is no associated magnetic disturbance up to the limit of the catalogue (30′ in D, or 150γ in H or V). In the case of spot No. 5589, mean area 1838 millionths, the magnetic traces show no appreciable disturbance for several days before or after the C.M.P. of the spot.

TABLE V.

The Largest Sunspots (*Mean Area* > 1500 *Millionths*) 1874–1927.

No.	Year.	Mean Area.	C.M.P.	Magnetic Storm.
9861	1926	3285	Jan. 24·5	Jan. 22–23 Jan. 26–27 G
9830	1925	2587	Dec. 28·3	Dec. 27–29
5441	1905	2500	Feb. 4·2	Feb. 3–4
2421	1892	2402	Feb. 11·9	Feb. 13–14 G
8181	1917	2389	Aug. 10·1	Aug. 9–10 G Aug. 13–15 G
5682	1905	2336	Oct. 20·2	
7977	1917	2176	Feb. 9·5	
4497	1897	2132	Jan. 9·4	
9143	1920	1966	Mar. 21·6	Mar. 22–23 G
6205	1907	1902	June 19·7	
10041	1926	1881	Sept. 19·7	Sept. 19–22
5589	1905	1838	July 16·7	

TABLE V.—*continued.*

No.	Year.	Mean Area.	C.M.P.	Magnetic Storm.
5473	1905	1762	Mar. 7·9	
729	1882	1744	Apr. 19·1	Apr. 16–17 G Apr. 20 G
4428	1896	1731	Sept. 16·7	Sept. 17–18 Sept. 20
5093	1903	1726	Oct. 11·6	Oct. 12–14
885	1882	1711	Nov. 18·9	Nov. 17–20 G
6108	1907	1619	Feb. 12·4	Feb. 9–10 G
3106	1893	1594	Aug. 7·4	Aug. 6–8
2581	1892	1591	July 10·2	July 12–14 G
9885	1926	1579	Mar. 3·8	Mar. 5–6
4781	1898	1556	Sept. 9·2	Sept. 9 G
9108	1920	1526	Jan. 28·0	

8. *Summary.*—A comparison between the large magnetic storms and large sunspots during the period 1874 to 1927 shows that individual storms and individual spots are associated with each other more often than can be ascribed to mere chance. The tendency to association appears to be greater for the very largest storms of all. It would seem that whatever solar activity is responsible for a magnetic storm, if it is sufficient to produce an unusually large storm, it will probably manifest itself as a large sunspot. But attention has been called to a case of a very large storm taking place at a time when only very moderate spots were visible.

No very definite evidence has been found of a tendency for these large storms to be followed by another magnetic disturbance after an interval of one solar rotation. The evidence available suggests that there may be a slight tendency for recurrence, but it is not very conclusive.

Royal Observatory, Greenwich :
1928 *May* 8.

On the Law of Stellar Distribution derived from Proper Motions.

By W. M. Smart, M.A., D.Sc.

(*Communicated by the Director, Cambridge Observatory.*)

§ 1. *Summary.* (*a*) Dyson's proposed law of stellar distribution, namely,

$$D(r) = \frac{A}{r} e^{-h^2k^2r^2}.$$

is the basis of the analysis.

(*b*) The proper motion statistics in one of the regions recently surveyed at Cambridge are compared with the theoretical implications

of the assumed density law by three methods. In the first, the distribution of proper motions (according to size) in a selected position angle sector is considered. The theoretical distribution depends on the particular values of the drift constants in the region as well as on the form of $D(r)$. In the second method, we are concerned with the various forms of frequency curves obtained when we consider only proper motions exceeding, say, 2″, 3″, etc. per century. In the third method, we consider, irrespective of position angle, the distribution of all the proper motions in the region.

(*c*) The third method is applied to all the proper motions in different parts of the sky lately measured at Cambridge.

(*d*) The investigation appears to indicate that the value of k in the assumed law $D(r)$ increases with galactic latitude in accordance with the known spheroidal distribution of the stars.

§ 2. The problem of determining the law of distribution of the stars in space is usually attempted from a statistical study of the numbers of stars of different apparent magnitudes. Owing to the immense range in stellar luminosities indicated by a ratio of at least 1,000,000 to 1, or by a difference of absolute magnitude not less than 15, some special hypothesis must be made at the outset, for example, that any given volume of space contains the same proportion of stars of a particular luminosity as any other volume. The space velocities of the stars, on the other hand, show a dispersion very much less than that in luminosity, and for this reason statistical proper motions (assuming these to be known right up to the limits of the stellar universe) afford a more satisfactory criterion of stellar distances than the observed apparent magnitudes. At the present time, however, we have very little information concerning proper motions of stars fainter than the twelfth magnitude, and consequently the study of proper motions enables us to penetrate into the stellar universe but a little further than the region already explored by the direct methods of parallax measurements. In the near future we may expect to know the proper motions of stars down to the 14th or 15th magnitude in representative areas of the sky, and then the methods described in the present paper may be given a more exhaustive trial than it is possible now to give with the rather limited observational data at present collected.

§ 3. One of the pioneer investigations into the distribution of the stars by means of proper motions is that of Sir Frank Dyson published * in 1913. If v is the linear velocity of a star in a particular co-ordinate, then on both Eddington's and Schwarzschild's theories of stellar motions, the velocity distribution is assumed, in the absence of systematic motion, to be given by the Maxwellian Law $e^{-h^2v^2}$. The number of stars between distances r and $r + dr$ being denoted by $f(r)dr$, the number of proper motions of these stars between μ and $\mu + d\mu$ is proportional to $rf(r)e^{-h^2r^2\mu^2}d\mu dr$. Dyson compared this formula with the observed distribution of proper motions (or rather of the components of proper motion in a certain direction) of the 3735 stars in Carrington's Circum-

* *M.N.*, **73**, 334 and *M.N.*, **73**, 402. See also A. S. Eddington, *M.N.*, **73**, 346.

polar Catalogue. The stars in this catalogue are within 9° of the north pole. The components of proper motion projected along the great circle from the pole towards right ascension 0^h are at right angles to the direction of the solar motion, and very nearly at right angles to the directions of both streams; thus the linear velocities projected along this great circle ought to give the random distribution contemplated. In the first paper quoted, Dyson showed that the distribution law

$$f(r) = re^{-\kappa^2 r^2}$$

accounted very satisfactorily for the observed distribution of the proper motion components.

In the second paper, Dyson applied this formula to the components of proper motion projected along the great circle from the pole towards right ascension 6^h, that is in the direction of the solar and stream motions. The distribution of linear velocities in this direction follows the law $e^{-h^2(v-V)^2}$ where V is the linear stream or drift velocity. Dyson's investigations thus deal with a special region of the sky in which the proper motions can be separated out into two directions, one giving a random distribution of the linear velocities, the other giving a random distribution superimposed on a common drift motion. The object of the present paper is to obtain a general theory applicable to any region in the sky and to the distribution of proper motions in any position angle, and to compare the theoretical distributions of the proper motions with the observed distributions in a particular region.

§ 4. Consider N stars forming a drift in any limited region of the sky, subtending a solid angle S. Let w denote the random linear velocity of a star projected on the tangent plane of the centre of the region. Let V denote the linear drift velocity projected on the same plane; the projected drift velocity will, of course, be in a particular direction on the tangent plane. Let θ denote the angle between any other position angle and that in which the drift velocity is directed. Then the number of stars with linear velocities between w and $w + dw$ and in directions between θ and $\theta + d\theta$ is

$$\frac{Nh^2}{\pi} d\theta w dw e^{-h^2(w^2+V^2-2Vw\cos\theta)},$$

in which h is a constant simply related to the mean random space velocity of the N stars.* Let $D(r)$ denote the density function, that is, the number of stars in unit volume at the distance r; this distribution refers to stars brighter than a particular limiting magnitude—in the present application this limiting magnitude is between 11 and 12. Then the element of the cone (through the boundary of the region concerned) between r and $r + dr$ has a volume Sr^2dr and therefore contains

$$Sr^2D(r)dr \text{ stars.}$$

* See A. S. Eddington, *M.N.*, **68**, 588.

Of these stars, the number with projected linear velocities between w and $w + dw$ and in directions between θ and $\theta + d\theta$ is

$$\frac{Sh^2}{\pi} r^2 D(r) w e^{-h^2(w^2+V^2-2Vw\cos\theta)} dr d\theta dw.$$

But if μ is the proper motion,

$$w = r\mu.$$

Hence the number of stars, in the element of volume at a distance r, with proper motions between μ and $\mu + d\mu$ and in directions between θ and $\theta + d\theta$ is

$$\frac{Sh^2}{\pi} r^4 D(r) \mu e^{-h^2(r^2\mu^2+V^2-2Vr\mu\cos\theta)} d\mu d\theta dw.$$

Let $M(\mu, \theta) d\mu d\theta$ denote the number of stars in the whole cone with proper motions between μ and $\mu + d\mu$ and in directions between θ and $\theta + d\theta$. Then

$$M(\mu, \theta) d\mu d\theta = \frac{Sh^2}{\pi} \mu d\mu d\theta \int_0^\infty r^4 D(r) e^{-h^2(r^2\mu^2+V^2-2Vr\mu\cos\theta)} dr. \qquad (1)$$

In any region, where there is but a single drift of stars, the function $M(\mu, \theta)$ gives the distribution of proper motions in any direction θ; if this function can be determined from observations, (1) is then an integral equation in $D(r)$ which can theoretically now be determined. But, in general, each region of the sky shows two drifts, and the evaluation of $D(r)$ from the observed distribution of proper motions becomes increasingly difficult. We shall adopt Dyson's density function in the theory and see how far it is consistent with the observed distributions of proper motions.

§ 5. Assume then that

$$D(r) = \frac{A}{r} e^{-h^2k^2r^2} \quad . \quad . \quad . \quad . \qquad (2)$$

which, being an expression for the volume density, is essentially of the same form as Dyson's function $f(r)$ in § 3 which is the expression for the linear density. The total number of stars in the cone is given by

$$N = \int_0^\infty SD(r) r^2 dr = AS \int_0^\infty r e^{-h^2k^2r^2} dr$$

from which

$$AS = 2h^2k^2N \quad . \quad . \quad . \quad . \qquad (2a)$$

Inserting (2) in (1) we have

$$M(\mu, \theta) = \frac{ASh^2}{\pi} \mu \int_0^\infty r^3 e^{-h^2[(\mu^2+k^2)r^2+V^2-2V\mu r\cos\theta]} dr \quad . \qquad (3)$$

Making the substitutions

$$\left.\begin{aligned} h^2(\mu^2+k^2) &\equiv a^2 \\ a\left(r-\frac{\mathrm{V}\mu\cos\theta}{\mu^2+k^2}\right) &\equiv x \\ \frac{h\mathrm{V}\mu\cos\theta}{(\mu^2+k^2)^{\frac{1}{2}}} &\equiv y \end{aligned}\right\} \quad . \quad . \quad . \quad (4)$$

we find that

$$\mathrm{M}(\mu,\theta) = \frac{\mathrm{AS}h^2\mu}{\pi a^4}e^{-h^2\mathrm{V}^2}e^{y^2}\int_{-y}^{\infty}(x+y)^3e^{-x^2}dx \quad . \quad (5)$$

Let

$$\mathrm{I}(y) = \int_{-y}^{\infty}e^{-x^2}dx \quad . \quad . \quad . \quad . \quad (6)$$

Expanding $(x+y)^3$ under the integral in (5) and making use of the following :—

$$\int_{-y}^{\infty}xe^{-x^2}dx = \tfrac{1}{2}e^{-y^2},$$

$$\int_{-y}^{\infty}x^2e^{-x^2}dx = \tfrac{1}{2}\mathrm{I}(y) - \tfrac{1}{2}ye^{-y^2},$$

$$\int_{-y}^{\infty}x^3e^{-x^2}dx = \tfrac{1}{2}y^2e^{-y^2} + \tfrac{1}{2}e^{-y^2},$$

we obtain

$$\mathrm{M}(\mu,\theta) = \frac{\mathrm{AS}h^2}{\pi a^4}\mu e^{-h^2\mathrm{V}^2}e^{y^2}[(y^3+\tfrac{3}{2}y)\mathrm{I}(y) + (\tfrac{1}{2}y^2+\tfrac{1}{2})e^{-y^2}] \quad (7)$$

The function $\mathrm{M}(\mu,\theta)$ gives the distribution of proper motions in any direction θ. Although the numerical values of the functions on the right-hand side of (7) can be obtained easily enough, the application of this formula entails an excessive amount of laborious calculation. We therefore proceed as follows.

Let $\mathrm{N}(\mu,\theta)d\theta$ now denote the number of stars with proper motions between μ and ∞ in directions between θ and $\theta+d\theta$. Then

$$\mathrm{N}(\mu,\theta) = \int_{\mu}^{\infty}\mathrm{M}(\mu,\theta)d\mu \quad . \quad . \quad . \quad (8)$$

From (4) we have

$$y^2 = h^2\mathrm{V}^2\cos^2\theta\left(1-\frac{k^2}{\mu^2+k^2}\right),$$

so that

$$ydy = h^2\mathrm{V}^2\cos^2\theta\,.\,\frac{k^2}{(\mu^2+k^2)^2}\mu d\mu$$

or

$$\frac{\mu d\mu}{a^4} = \frac{ydy}{h^6k^2\mathrm{V}^2\cos^2\theta}.$$

Let us write

$$h\mathrm{V}\cos\theta = \tau \quad . \quad . \quad . \quad . \quad (9)$$

Then when $\mu = \infty$, $y = \tau$.

We then obtain from (7) and (8),

$$N(\mu, \theta) = \frac{ASe^{-h^2V^2}}{\pi h^2 k^2 \tau^2} \int_y^r [(y^4 + \tfrac{3}{2}y^2)e^{y^2}I(y) + (\tfrac{1}{2}y^3 + \tfrac{1}{2}y)]dy \qquad (10)$$

Integrating the first part of the integral (denoted by K) on the right-hand side of (10) by parts, we have

$$K \equiv \int (y^4 + \tfrac{3}{2}y^2)e^{y^2}I(y)dy = \tfrac{1}{2}e^{y^2}(y^3 + \tfrac{3}{4}y)I(y) - \tfrac{1}{2}\int e^{y^2}\frac{d}{dy}[(y^3 + \tfrac{3}{2}y)I(y)]dy.$$

Now

$$I(y) = \int_{-y}^{\infty} e^{-x^2}dx\,; \quad \text{therefore} \quad \frac{dI(y)}{dy} = e^{y^2}.$$

K then becomes

$$K = \tfrac{1}{2}e^{y^2}(y^3 + \tfrac{3}{2}y)I(y) - \tfrac{1}{8}(y^4 + 3y^2) - \tfrac{1}{2}\int e^{y^2}(3y^2 + \tfrac{3}{2})I(y)dy.$$

Also

$$e^{y^2}(3y^2 + \tfrac{3}{2})I(y) = \frac{d}{dy}[\tfrac{3}{2}ye^{y^2}I(y) - \tfrac{3}{4}y^2].$$

K then reduces to the simple expression given by

$$K = \tfrac{1}{2}y^3e^{y^2}I(y) - \tfrac{1}{8}y^4,$$

so that the complete integral on the right of (10) becomes

$$\tfrac{1}{2}y^3e^{y^2}I(y) + \tfrac{1}{4}y^2$$

and

$$N(\mu, \theta) = \frac{ASe^{-h^2V^2}}{2\pi h^2 k^2 \tau^2}[\tau^2\{\tau e^{\tau^2}I(\tau) + \tfrac{1}{2}\} - y^2\{ye^{y^2}I(y) + \tfrac{1}{2}\}].$$

Let $f(y)$ be defined by

$$\frac{\sqrt{\pi}}{2}f(y) = ye^{y^2}I(y) + \tfrac{1}{2}.$$

Then

$$N(\mu, \theta) = \frac{ASe^{-h^2V^2}}{4\sqrt{\pi}h^2k^2}\left\{f(\tau) - \frac{y^2}{\tau^2}f(y)\right\} \quad . \quad . \quad (11)$$

The function $f(y)$ is that introduced in Professor Eddington's theory.*

By (9), $\tau = hV\cos\theta$. Let us generalise and assume that the drift is pointing in the direction of position angle θ_1. Then we write

$$\tau = hV\cos(\theta - \theta_1) \quad . \quad . \quad . \quad (12)$$

and the formula (11) then refers to position angle θ.

Write

$$\eta = \frac{\mu}{\sqrt{\mu^2 + k^2}} \quad . \quad . \quad . \quad . \quad (13)$$

* *M.N.*, **68**, 588.

so that by (4) and (12),

$$y = \eta\tau.$$

By (13), η is not greater than 1. Let θ' be defined by

$$\cos(\theta' - \theta_1) = \eta\cos(\theta - \theta_1) \quad . \quad . \quad . \quad (14)$$

so that

$$y = h\mathrm{V}\cos(\theta' - \theta_1).$$

Introduce the function given by

$$\phi(h\mathrm{V}, \theta - \theta_1) = e^{-h^2\mathrm{V}^2} f(\tau).$$

I have tabulated the values of this function in *Monthly Notices*, **87**, 127 for various values of $h\mathrm{V}$ and at steps of 5° in the values of $\theta - \theta_1$. We then obtain the formula giving the number of stars with proper motions greater than μ and in any position angle θ, namely,

$$\mathrm{N}(\mu, \theta) = \frac{\mathrm{AS}}{4\sqrt{\pi}h^2k^2}\{\phi(h\mathrm{V}, \theta - \theta_1) - \eta^2\phi(h\mathrm{V}, \theta' - \theta_1)\}.$$

But from (2*a*), $\mathrm{AS} = 2h^2k^2\mathrm{N}$. Hence we have, finally,

$$\mathrm{N}(\mu, \theta) = \frac{\mathrm{N}}{2\sqrt{\pi}}\{\phi(h\mathrm{V}, \theta - \theta_1) - \eta^2\phi(h\mathrm{V}, \theta' - \theta_1)\} \quad . \quad (15)$$

In any region of the sky there are in general two drifts, each with its characteristic constants N, $h\mathrm{V}$, and θ_1. We have thus to compare the observed data with the combined theoretical results derived from two equations of the type (15).

§ 6. The analysis of the previous section breaks down when the value of k is zero. Then the density function is given by

$$\mathrm{D}(r) = \frac{\mathrm{A}}{r}.$$

Let N be the number of stars within a cone of solid angle S and extending to a distance R. Then

$$\mathrm{N} = \mathrm{AS}\int_0^{\mathrm{R}} r dr,$$

so that

$$\mathrm{AS} = \frac{2\mathrm{N}}{\mathrm{R}^2}.$$

The formulæ (4) become

$$a = h\mu; \qquad x = h\mu r - \tau; \qquad y = \tau,$$

and $\mathrm{M}(\mu, \theta)$ is now given by

$$\mathrm{M}(\mu, \theta) = \frac{\mathrm{AS}}{\pi h^2}\frac{e^{-h^2\mathrm{V}^2}}{\mu^3}e^{\tau^2}\int_{-\tau}^{h\mu\mathrm{R}-\tau}(x + \tau)^3 e^{-x^2}dx,$$

or if R is large, we can write this equation with sufficient accuracy,

$$M(\mu, \theta) = \frac{AS}{\pi h^2} \frac{e^{-h^2V^2}}{\mu^3} e^{\tau^2} \int_{-\tau}^{\infty} (x + \tau)^3 e^{-x^2} dx.$$

The integral is now simply a function of τ, so that $M(\mu, \theta)$ is proportional to μ^{-3}. Let us write

$$M(\mu, \theta) = \frac{CF(\tau)}{\mu^3}$$

where C is a constant. $F(\tau)$ has the value given by

$$F(\tau) = (\tau^2 + \tfrac{3}{2})\phi(hV, \theta - \theta_1) - \frac{1}{2\sqrt{\pi}} e^{-h^2V^2}.$$

The number $N(\mu, \theta)$ of stars with proper motions greater than μ in position angles between θ and $\theta + d\theta$ is then given by

$$N(\mu, \theta) = \frac{2CF(\tau)}{\mu^2} \quad . \quad . \quad . \quad . \quad (16)$$

We may regard (16) as the limiting form of (15).

§ 7. The preceding theory has been applied tentatively to the photographic proper motions, lately derived at Cambridge, of 666 stars in a region of the sky around $0^h\ 15^m$, $+ 50°$. This region includes the three photographic regions of λ Andromedæ, η Cassiopeiæ, and Groombridge 34, which form Group I. in my paper * on "The Constants of the Star Streams," together with the region of β Cassiopeiæ containing 121 stars whose proper motions have more recently been measured. The analysis of the photographic proper motions shows that the stars are practically equally divided between the two streams; we shall accordingly assume that the number of stars belonging to Drift I. in the region with which we are concerned is equal to the number of stars belonging to Drift II. We shall adopt the following values for the drift constants (see the paper to which reference has just been made):—

Drift I. : position angle	$(\theta_1) = 100°$;	drift velocity	$(hV_1) = 1{\cdot}5$
Drift II. : "	" $(\theta_2) = 200°$;	"	" $(hV_2) = 0{\cdot}7$

§ 8. In the principal formula (15), the values of η and η^2 are required, the former for computing the values of the subsidiary angle $(\theta' - \theta_1)$ defined by equation (14). η is of course a function of μ and k—equation (13)—and it is a simple matter to compute the values of η and η^2 for different values of μ ($0''{\cdot}0$, $1''{\cdot}0$, . . . $8''{\cdot}0$ per century) and for sufficiently representative and sensitive values of k.

§ 9. There are three methods of comparing the observed distributions of proper motions with the theoretical distributions which are a consequence of the assumed law of stellar density. These methods will be denoted by A, B, and C.

* *M.N.*, 87, 122.

In method A we consider a definite position angle θ or in practice a sector between $\theta - 5°$ and $\theta + 5°$. The observations furnish the number of stars moving in this sector with proper motions between 0″·0 and 0″·5 per century, 0″·5 and 1″·0, and so on. After smoothing this distribution we then obtain the total number $N(\mu, \theta)$ of stars in the sector with proper motions greater than any particular value of μ. For a series of values of μ, we derive a curve which can be compared with a series of the theoretical curves based on equation (15) for different values of the constant k.

If the assumed density law holds for this particular distribution, it is then possible to arrive at a value of k giving a good fit between the observed and theoretical curves. Other sectors can be treated in the same way, and the test of the assumed density law will then depend on the consistency (or otherwise) of the series of values of k derived from the various sectors.

In method B, we consider the observed values of $N(\mu, \theta)$, for a particular value of μ and for all values of θ. If we take μ to be 0″·0 then we are concerned with all the proper motions measured and we obtain the frequency curves from which the stream constants have been derived (see fig. 1, *M.N.*, **87**, 130). When $\mu = 2''·0$, for example, we obtain a frequency curve of proper motions exceeding 2″·0 per century; in general this curve will be different from that obtained from the value 0″·0. The equation (15) furnishes the means of deriving a theoretical frequency curve, depending on the particular value of k employed, of the proper motions exceeding any given value of μ. If the assumed density law is obeyed, it should then be possible to pick out from a series of theoretical curves (for different values of k) one that fits the observed frequency curve.

The third method C depends on the total number of stars in the region with proper motions exceeding any particular value of μ. This number can then be compared with the theoretical number given by

$$\int_0^{2\pi} N(\mu, \theta) d\theta,$$

μ being constant.

We shall now apply these methods to the 666 stars in the selected region. It must be said at once that the number of proper motions in this region is hardly sufficient to furnish a conclusive test of the theory. This part of the paper must then be regarded as a preliminary attempt to investigate the law of stellar distribution from the measured proper motions and as an experiment to indicate the lines on which a more thorough analysis may be made when a particular region of the sky has been more extensively surveyed. Finally, we shall apply method C to all the stars whose proper motions have been obtained at Cambridge, over 3000 in number.

§ 10. We shall now briefly consider method A, taking as an example the distribution of proper motions in the sector 115°–125°. The observations furnish us with the number of stars in this sector with centennial proper motions between 0″·0 and 0″·5, 0″·5 and 1″·0, etc. These numbers

are represented graphically in fig. 1. The curve has been smoothed as seemed consistent with the data, and from this smoothed curve we derive the number of stars in the sector with centennial proper motions exceeding 2″, 3″, . . . 8″. These numbers are represented by the full-line curve in fig. 2 (a factor has been applied so as to make the number of stars, with proper motions of 2″ per century and over, equal to 50). We now compare this curve with the theoretical curves, for different values of the constant k, derived from equation (15). We know the values of the constants for Drifts I. and II., and it is then a straightforward piece of computing to obtain for Drift I.—and afterwards for Drift II.—the numerical values of $N(\mu, \theta)$ for this sector and for different values of μ. The combination of the two sets of numbers

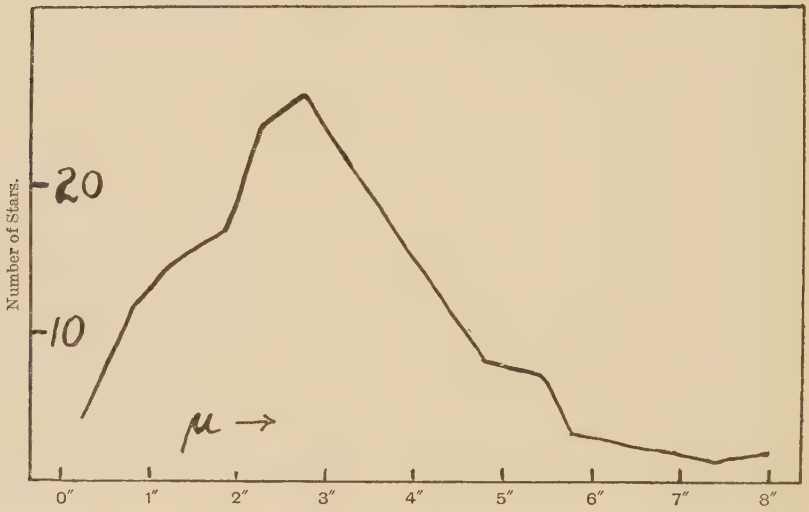


FIG. 1.

for the two drifts leads to the theoretical curves of fig. 2 after the application of a suitable factor as in the case of the observed curve. It is seen that for this sector a fairly good fit between theory and observation is obtained if k is about 0·2. Six other sectors between 80° and 200° were examined in a similar way. Four of these gave values of k ranging between 0·2 and 1·0, and in the remaining two sectors the observed curves did not conform to the implications of the assumed distribution law $D(r)$. Owing to the predominance of Drift I., the numbers of stars in the sectors fall off rapidly on either side of position angle 100°, and it could hardly be expected that a conclusive test of this method would follow from the small number of stars available. It may be added by way of comparison that the value of k emerging from Dyson's investigation of 3700 stars was 1·2.

§ 11. An attempt has been made to correct the statistical data for the influence of the errors of observation. The effect of an error in

the proper motion of a star is twofold. There is, firstly, the change in the total proper motion μ, and, secondly, the change in position angle θ. The effect of the latter may be to switch the star from a particular 10° sector into an adjacent sector ; and the effect is of course greatest for small values of μ. Professor Eddington has given an excellent formula * for correcting statistics depending on one variable. In the

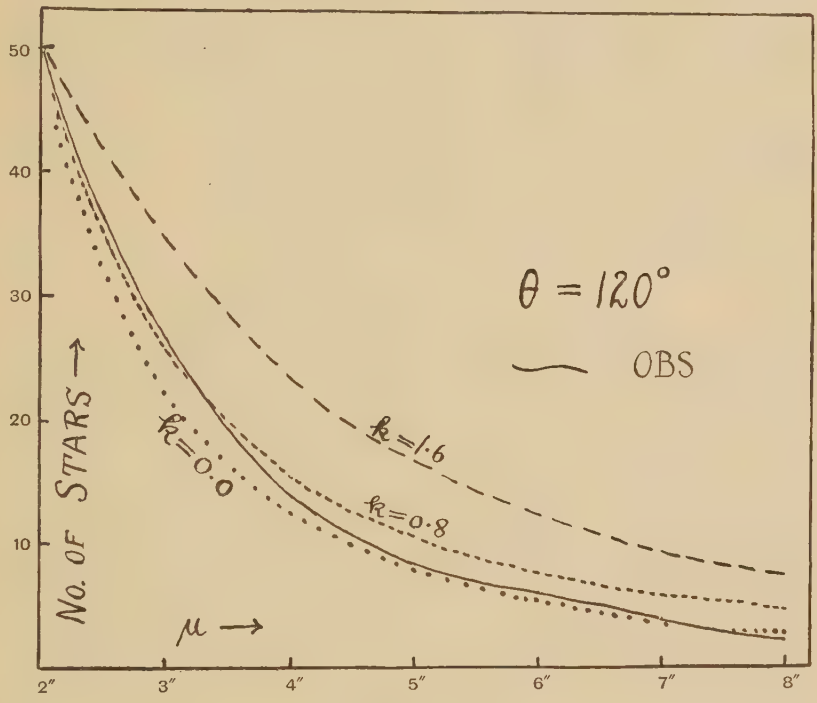


Fig. 2.

problem under consideration, we are dealing with a statistical array involving two variables μ and θ.

Let x, y denote the rectangular components of the proper motion of a star. Let

$$u(x, y)dxdy$$

be the number of stars with proper motion components observed between x and $x + dx$, y and $y + dy$. Let

$$v(x, y)dxdy$$

be similarly the true number.

We can take the probable error for the components x and y to be the

* *M.N.*, **73**, 359.

same; denote it by $\frac{0\cdot477}{H}$. If α and β denote the errors in x and y respectively for a particular star, then, following Eddington, we have

$$u(x, y) = \frac{H^2}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v(x+\alpha, y+\beta)e^{-H^2\alpha^2-H^2\beta^2}d\alpha d\beta$$

$$= \frac{H^2}{\pi}\int_{-\infty}^{\infty} e^{\alpha\frac{\partial}{\partial x}-H^2\alpha^2}d\alpha\int_{-\infty}^{\infty} e^{\beta\frac{\partial}{\partial y}-H^2\beta^2}v(x, y)d\beta$$

$$= e^{\frac{1}{4H^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)}v(x, y)$$

from which

$$v(x, y) = e^{-\frac{1}{4H^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)}u(x, y).$$

Transform to polar co-ordinates (μ, θ). Then

$$v(\mu, \theta) = e^{-\frac{1}{4H^2}\left(\frac{1}{\mu}\frac{\partial}{\partial\mu}+\frac{\partial^2}{\partial\mu^2}+\frac{1}{\mu^2}\frac{\partial^2}{\partial\theta^2}\right)}u(\mu, \theta).$$

Let the function be tabulated at intervals a of μ and at intervals b of $\mu\theta$. Then neglecting third and higher differences, we obtain

$$v(\mu, \theta) = u(\mu, \theta) - \frac{1}{4H^2}\left\{\frac{1}{\mu}\frac{u_1(\mu)}{a}+\frac{u_2(\mu)}{a^2}+\frac{u_2(\theta)}{b^2}\right\}+\frac{1}{2}\frac{1}{(4H^2)^2}\cdot\frac{1}{\mu^2}\frac{u_2(\mu)}{a^2},$$

where the suffixes indicate first and second differences. In practice a will be constant; in fig. 2 we have taken a to be $0''\cdot5$. We have also taken the intervals in θ to be 10°, that is, in circular measure, $\frac{1}{6}\frac{1}{3}$ approximately. The interval b increases with μ. Denote μ by na. Then the interval $b = \frac{1}{6}\frac{1}{3}na$.

If ϵ is the probable error for the components x and y of the proper motions

$$\frac{1}{2H} = 1\cdot05\epsilon$$

and our formula finally becomes

$$v(\mu, \theta) = u(\mu, \theta) - \left(\frac{1\cdot05\epsilon}{a}\right)^2\left[\frac{1}{n}u_1(\mu)+u_2(\mu)\left\{1-\frac{1}{2}\left(\frac{1\cdot05\epsilon}{\mu}\right)^2\right\}+\left(\frac{63}{113n}\right)^2u_2(\theta)\right].$$

As might be expected from the comparatively small number of stars in the region of the sky examined, the statistical corrections could hardly be regarded with any great measure of confidence. The real trouble, if we are considering only stars with centennial proper motions of $2''$ and over, is that we are unable to obtain from the smoothed curve of fig. 1 really reliable values of the second difference $u_2(\mu)$ in the neighbourhood of $\mu = 2''$. If we had had a very much larger number of stars to deal with, we could have considered only proper motions of $3''$ per century and over, with possibly the disappearance of uncertainty regarding the values of $u_2(\mu)$. It is hardly worth while,

under these circumstances, giving any detailed results. The general effect, however, appeared to be that the constant k in the density function was somewhat diminished.

§ 12. We discuss briefly method B, giving as before a single example. When we consider the proper motions of all stars in a region we obtain a frequency curve of the type illustrated in *M.N.*, **87**, 122. If we restrict the stars to those with proper motions, say, of 2″ per century and over, equation (15) shows that the frequency curve ought to depend on the value of the constant k occurring in the assumed law of distribution. Fig 3 shows two such frequency curves derived from (15)

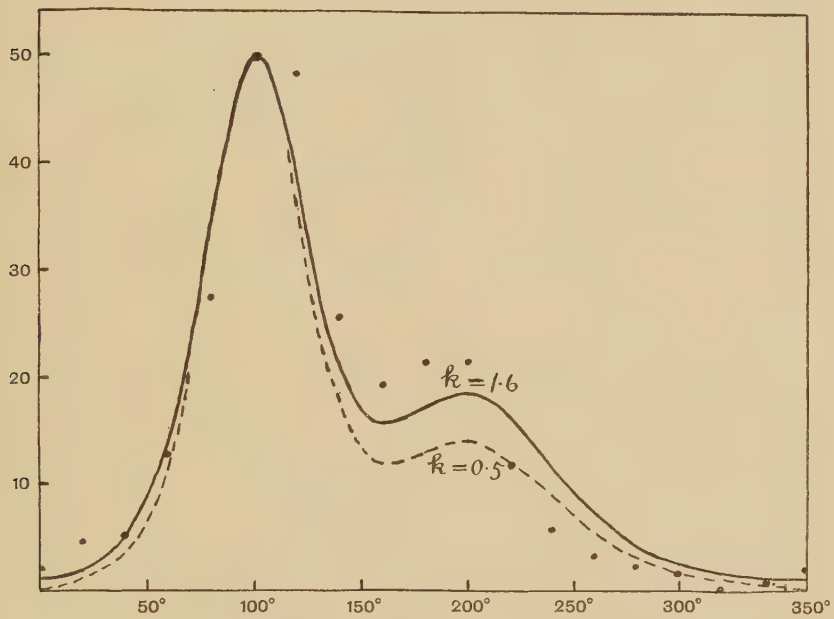


Fig. 3.

for the region we are considering, the values of k being 1·6 and 0·5. The scale has been so chosen as to make the maxima of the two curves each equal to 50. The theoretical frequency curves are evidently sensitive to changes in the value of k. The dots in the figure represent the observed distribution. In the region there are only 348 stars with proper motions of 2″ per century and over, and these, of course, are spread over all the position angles between 0° and 360°. Again, the material is not nearly extensive enough to make a reliable test of the theory. But it is well to emphasise the dependence, generally, of the forms of the frequency curves, derived from selected proper motions, on the law of distribution.

§ 13. In the third method of testing the assumed law of stellar distribution, we are concerned with the total number of stars in the

region with proper motions exceeding a certain value. Let $R(\mu)$ denote the number of stars in the whole region, belonging to one drift, with centennial proper motions of μ and over; then evidently from equation (15)

$$R(\mu) = \int_0^{2\pi} N(\mu, \theta) d\theta.$$

By equations (11) to (15) we can write $N(\mu, \theta)$ in the form given by

$$N(\mu, \theta) = \frac{N}{2\sqrt{\pi}} e^{-h^2V^2}\{f(\tau) - \eta^2 f(\eta\tau)\}$$

where N is the total number of stars in the drift.

If $\rho d\theta$ is the number of stars moving between position angles θ and $\theta + d\theta$, no restrictions being made to the magnitude of the proper motion, then

$$\rho d\theta = \frac{N}{2\sqrt{\pi}} e^{-h^2V^2} f(\tau) d\theta$$

so that

$$\int_0^{2\pi} \rho d\theta \equiv N = \frac{N}{2\sqrt{\pi}} e^{-h^2V^2} \int_0^{2\pi} f(\tau) d\theta$$

or

$$\int_0^{2\pi} f(\tau) d\theta = 2\sqrt{\pi} e^{h^2V^2}.$$

Similarly,

$$\int_0^{2\pi} f(\eta\tau) d\theta = 2\sqrt{\pi} e^{\eta^2 h^2 V^2}.$$

Hence

$$R(\mu) = N\{1 - \eta^2 e^{-h^2V^2(1-\eta^2)}\} \quad . \quad . \quad . \quad (17)$$

There is a similar equation for the other drift.

We compute the values of $R(\mu)$ for each drift for $\mu = 2'', 3'', \ldots 8''$ in succession and for different values of k. Fig. 4 shows the theoretical curves derived in this way, for $k = 1{\cdot}6$ and $k = 0{\cdot}0$. The scale is chosen so that the total theoretical number of stars with centennial proper motions of $2''$ and over is equal to 50. The theoretical curves can now be compared with the observed curve derived from counts in the region (this is Group I.). It is seen that observation and theory are in close accord if the value of k is about $0{\cdot}3$. Method C appears then to give a definitive result; the proper motion data are in agreement with the assumed law of stellar distribution and the value of the constant k is about $0{\cdot}3$.

§ 14. From equation (17) it is seen that the function $R(\mu)$ for any region of the sky depends on the values of the stream constants for that region. The form of the theoretical curves, such as are shown in fig. 4, will thus vary from one part of the sky to another, except in the limiting case when $k = 0{\cdot}0$. When we deal with regions scattered over the sky, we should strictly take account of this fact in contrasting

theory with observation. We now apply the theory to the 3029 stars in the eight groups of regions * photographed at Cambridge. The drift constants which concern us are the velocities hV_1, hV_2 and the numbers N_1, N_2 of the stars in the drifts. From the results of the paper quoted, N_1 can be taken equal to N_2. Also in the majority of regions, the observed value of hV_1 is not very different from 1·5 (the value in the region we have been particularly considering); and this

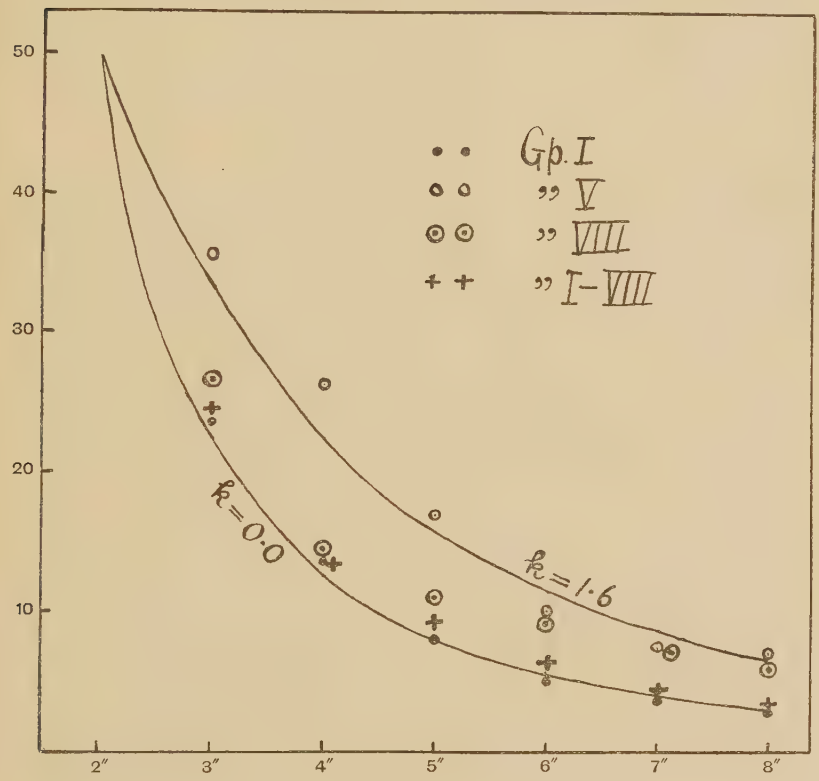


FIG. 4.

also applies with greater force to the value of hV_2, viz. 0·7. It is thus possible without calculating the values of $R(\mu)$ for the individual groups of regions to obtain a fairly accurate idea of the possible agreement of observation with theory. In Table III., *M.N.*, **87**, 125 I have given the numbers of stars in these regions with centennial motions between various limits. From the data, we obtain in the same way as for the single region we have been discussing more particularly, a series of values of $R(\mu)$ which are represented by crosses in fig. 4. In this figure the observed values indicate a value of k of about 0·3

* *M.N.*, **87**, 122.

or 0·4. The assumed density law is in fact very well represented by the observations. When we consider the proper motions of 3″ per century and over, the curves point to a similar conclusion. In a recent memoir,* "Recherches sur les mouvements propres de 3802 étoiles," V. Nechvile concludes that for proper motions exceeding 2″ per century, the distribution is such that, in our notation,

$$\mathrm{R}(\mu) = \frac{\mathrm{C}}{\mu^2},$$

indicating that the density function D(r) is given by

$$\mathrm{D}(r) = \frac{\mathrm{A}}{r},$$

corresponding to the limiting value $k = 0{\cdot}0$. Our general results appear to indicate that k has a definite value, small no doubt, but of the order 0·3 or 0·4. Nechvile's results apply to the aggregate of regions scattered over the sky. We have already stated Dyson's result for k, viz. $k = 1{\cdot}2$, but this applies to a strictly localised region (within 9° of the pole). The general density law given by (2), namely,

$$\mathrm{D}(r) = \frac{\mathrm{A}}{r} e^{-h^2k^2r^2}$$

can hardly be expected, from what we know of stellar distribution, to furnish the identical value of k for every region on the celestial sphere. The local stellar cluster is a spheroidal distribution, and if the above density distribution is even a rough representation of the true distribution, we should expect the values of k to increase with galactic latitude. The great majority of the Cambridge regions are close to the galactic plane ; Nechvile's regions, however, have a wider distribution in galactic latitude. Our result for the eight Cambridge groups of regions ($k = 0{\cdot}3$ or $0{\cdot}4$) refers practically to the galactic plane. Two groups, however, are at considerable galactic latitudes, Group V. with a total number of 126 stars in galactic latitude + 62° and Group VIII. with 256 stars in galactic latitude − 31°. The observed values of R(μ) in each of these regions—$\mu = 2''$, $3''$, . . . per century—are plotted in fig. 4. In Group V., the drift velocities are almost the same as those taken in our calculations, viz. 1·5 and 0·7 ; in Group VIII. the differences are rather larger since $h\mathrm{V}_1 = 1{\cdot}3$ and $h\mathrm{V}_2 = 0{\cdot}8$. However, these values for $h\mathrm{V}_1$ and $h\mathrm{V}_2$ in each region are sufficiently near the corresponding values on which the theoretical curves of fig. 4 are based to give a fair comparison between theory and observation. The data from Group V. (galactic latitude + 62°) appear to indicate a value of k in the neighbourhood of 1·6 ; while the data from Group VIII. (galactic latitude − 31°) appear to indicate a value in the neighbourhood of 1·0. This latter result is much the same as Dyson's result for the circumpolar stars in average galactic latitude + 28°. Although our data are not as extensive as we would wish, the evidence, such as

* *Publications de l'Observatoire National de Prague*, No. 4.

it is, points to an increase of k with galactic latitude in accordance with the known spheroidal-like distribution of the stars.

Proper motion statistics are inevitably bound up with the constants of the star-streams, and in deriving the law of stellar distribution this fact must be taken into account in the analysis. Dyson's law—although it neglects the possible and probable eccentric position of the sun in the local cluster—has much to recommend it on observational grounds. In any event, it appears to be a good first approximation.

Observatory, Cambridge:
1928 *May* 1.

The Correction of a Group of Parallax Observations and the Determination of the Main Series. By J. J. Nassau, Ph.D.

1. A method of correcting a series of parallaxes so that the negative parallaxes in the series will become positive has been considered by Sir Frank Dyson,* who has shown the value of such corrections in the case of a series of observations made under similar conditions. The seven hundred parallaxes of the Leander McCormick Observatory † offer an excellent example of such a series, for special precautions have been taken to make them homogeneous, as is evident from their statistical study.

One out of seven of these parallaxes is negative. The aim of this paper is to apply corrections to all the parallaxes so that negative parallaxes are removed, thus facilitating the determination of the main series and its probable spread.

2. Table I. gives the observed distribution, and in fig. 1 the smooth curve shows the adopted observed distribution. It has been shown by the writer ‡ that the probable error of each parallax of a series of parallaxes is better expressed by the mean probable error of the series, hence the mean value of $\pm 0''{\cdot}0096$ will be used for all the stars in the series.

If the smooth curve of fig. 1 is represented by $y=v(x)$, and Δx represents the interval of $0''{\cdot}005$ and Δy the mean value of successive increments of y, then the correction § $\frac{1}{2h^2}\frac{v'(x)}{v(x)}$ takes the approximate form

$$\frac{1}{2h^2}\frac{\frac{\Delta y}{\Delta x}}{y} = 0''{\cdot}040\frac{\Delta y}{y}.$$

---

* "A Method of Correcting Series of Parallax Observations," *M.N.*, **86**, 686.
† *Publications of the Leander McCormick Observatory*, vols. iii. and iv.
‡ "On the Probable Error of Stellar Parallaxes," *M.N.*, **88**, 441.
§ *M.N.*, **86**, 687.

TABLE I.

Limits of π.	No. of Observed π.	Limits of π.	No. of Observed π.
$''$ $<-''\cdot030$	1	$''\cdot040$ to $''\cdot045$	33
$-''\cdot030$ to $-''\cdot025$	3	$''\cdot045$,, $''\cdot050$	29
$-''\cdot025$,, $-''\cdot020$	7	$''\cdot050$,, $''\cdot055$	21
$-''\cdot020$,, $-''\cdot015$	7	$''\cdot055$,, $''\cdot060$	12
$-''\cdot015$,, $-''\cdot010$	17	$''\cdot060$,, $''\cdot065$	17
$-''\cdot010$,, $-''\cdot005$	27	$''\cdot065$,, $''\cdot070$	13
$-''\cdot005$,, $''\cdot000$	39	$''\cdot070$,, $''\cdot075$	15
$''\cdot000$,, $''\cdot005$	46	$''\cdot075$,, $''\cdot080$	12
$''\cdot005$,, $''\cdot010$	51	$''\cdot080$,, $''\cdot085$	6
$''\cdot010$,, $''\cdot015$	56	$''\cdot085$,, $''\cdot090$	6
$''\cdot015$,, $''\cdot020$	50	$''\cdot090$,, $''\cdot095$	5
$''\cdot020$,, $''\cdot025$	43	$''\cdot095$,, $''\cdot100$	5
$''\cdot025$,, $''\cdot030$	45	$''\cdot100$,, $''\cdot105$	3
$''\cdot030$,, $''\cdot035$	43	$''\cdot105$,, $''\cdot150$	14
$''\cdot035$,, $''\cdot040$	32	$''\cdot150$,, $<$	28

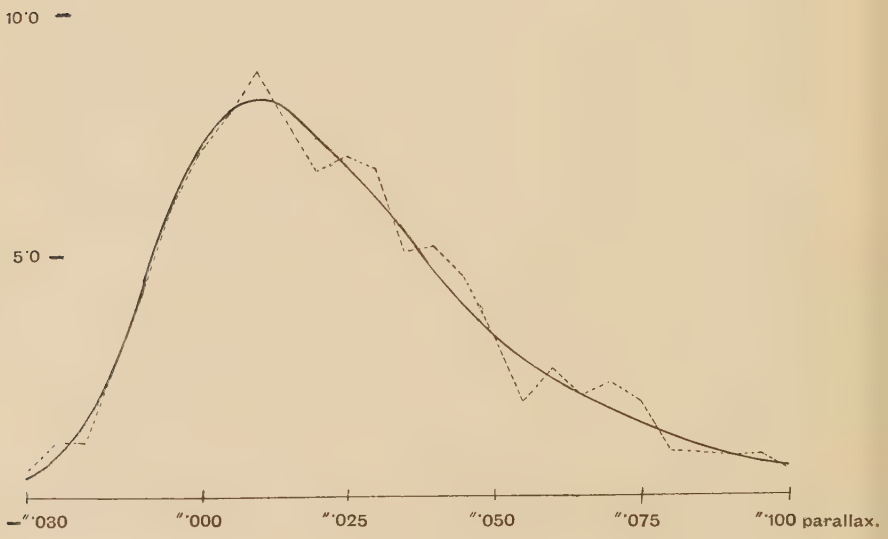


FIG. 1.

From this and the adopted distribution the corrections are computed and given in successive intervals of $0''\cdot005$ in Table II. The corrections clear practically all the negative parallaxes, and when the parallaxes are changed to absolute, all become positive. All parallaxes smaller than $0''\cdot009$ are increased; $0''\cdot010$ and $0''\cdot011$ receive zero correction, and all parallaxes above $0''\cdot011$ are decreased.

TABLE II.

π.	Correction.	π.	Correction.	π.	Correction.
$-''\cdot030$	$+''\cdot030$	$+''\cdot015$	$-''\cdot002$	$+''\cdot060$	$-''\cdot006$
$-\cdot025$	$+\cdot028$	$+\cdot020$	$-\cdot003$	$+\cdot065$	$-\cdot006$
$-\cdot020$	$+\cdot024$	$+\cdot025$	$-\cdot004$	$+\cdot070$	$-\cdot006$
$-\cdot015$	$+\cdot020$	$+\cdot030$	$-\cdot004$	$+\cdot075$	$-\cdot006$
$-\cdot010$	$+\cdot015$	$+\cdot035$	$-\cdot005$	$+\cdot080$	$-\cdot008$
$-\cdot005$	$+\cdot010$	$+\cdot040$	$-\cdot006$	$+\cdot085$	$-\cdot008$
$\cdot000$	$+\cdot005$	$+\cdot045$	$-\cdot007$	$+\cdot090$	$-\cdot007$
$+\cdot005$	$+\cdot003$	$+\cdot050$	$-\cdot007$	$+\cdot095$	$-\cdot007$
$+\cdot010$	$\cdot000$	$+\cdot055$	$-\cdot006$	$+\cdot100$	$-\cdot005$

The total positive correction was $2''\cdot008$.

The total negative correction was (this includes parallaxes up to $0''\cdot100$ only) $2''\cdot037$.

The true distribution is given in Table III., and is shown by the full line of fig. 2.

TABLE III.

Limits of π.	Number of Stars.	Limits of π.	Number of Stars.
$-''\cdot005$ to $''\cdot000$	2	$''\cdot050$ to $''\cdot055$	18
$\cdot000$,, $\cdot005$	13	$\cdot055$,, $\cdot060$	13
$\cdot005$,, $\cdot010$	150	$\cdot060$,, $\cdot065$	16
$\cdot010$,, $\cdot015$	109	$\cdot065$,, $\cdot070$	14
$\cdot015$,, $\cdot020$	65	$\cdot070$,, $\cdot075$	13
$\cdot020$,, $\cdot025$	57	$\cdot075$,, $\cdot080$	7
$\cdot025$,, $\cdot030$	42	$\cdot080$,, $\cdot085$	7
$\cdot030$,, $\cdot035$	38	$\cdot085$,, $\cdot090$	3
$\cdot035$,, $\cdot040$	42	$\cdot090$,, $\cdot095$	2
$\cdot040$,, $\cdot045$	17	$\cdot095$,, $\cdot100$	3
$\cdot045$,, $\cdot050$	20		

3. A comparison of this approximate method and a method given by Professor Eddington * concerning the true distribution of parallaxes might prove instructive.

The smooth curve AB of fig. 3 has for an equation $y=8\cdot1e^{-0\cdot2(x-0\cdot8)^2}$ and fits well the observed distribution which is shown by the broken line ; also $y=8\cdot2e^{-0\cdot038(x-0\cdot3)^2}$ represents well the observed distribution from B to C. The true distribution is given by the formula

$$Y = y - \left(\frac{1}{4h^2}\right)y^{II} + \frac{1}{2!}\left(\frac{1}{4h^2}\right)^2 y^{IV} - \ldots$$

The value of h is obtained from the average probable error (which in more convenient units is $\pm0\cdot96$). In applying this formula it was found

* *M.N.*, **73**, 359.

necessary to use up to the eighth derivative, and the two branches were connected by judicious averaging of points. The result is shown by the dotted curve of fig. 3.

It is interesting to note that 46 per cent. of the parallaxes so corrected lie between 0″·000 and 0″·020; 52 per cent. according to Dyson's

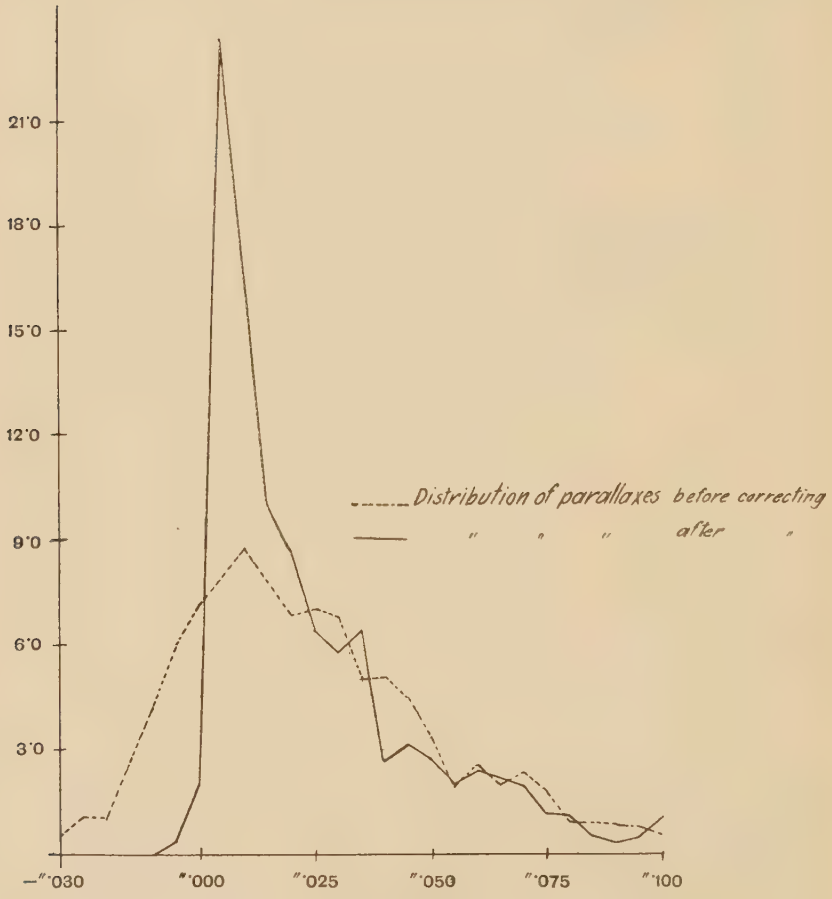


FIG. 2.

approximate method; and without any corrections only 20 per cent. lie between the same limits. It is difficult to explain the portion of the dotted curve of fig. 3 that lies below the zero line. It might be due to the fact that we are dealing with a finite portion of the curve only, or errors of fitting the observed distribution accumulate when the different terms of the correction were applied, or it might be due to the fact that the parallaxes used were relative.

4. *Corrected Absolute Magnitudes.*—To obtain the corrected absolute magnitude the relative parallaxes (as corrected in par. 3) of all the stars were reduced to absolute by applying the corrections given in volume iv., p. 280 (*Publications of the Leander McCormick Observatory*). Thus all the absolute corrected magnitudes were uniformly obtained. In the case of binary stars the average of the parallaxes was used in computing absolute magnitudes. Similar procedure was followed in

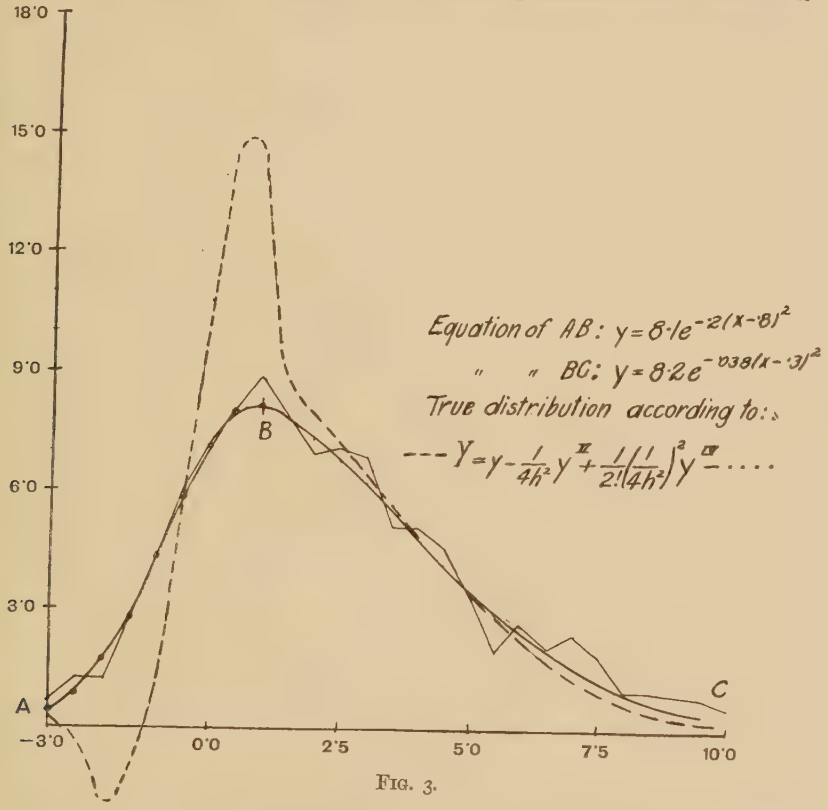


FIG. 3.

the case of the 21 stars of the Pleiades group. This was in accordance with the method used by Professor Mitchell.

Table IV. gives a summary of the results. The first column gives absolute magnitude for intervals of 0·3; the number under each spectral type indicates the number of stars within that interval. The hope of distinct separation between the giants and dwarfs was not achieved. This method of correcting is bound to give better individual parallaxes, but a marked improvement of a sub-group cannot be expected if the corrections derived from the whole group are applied. If the number of stars in each spectral class was sufficiently great to be dealt with separately, more satisfactory results would have been obtained.

TABLE

Also One Star for B5, −4·5 ;

	Oe5.	Bo.	B1.	B2.	B3.	B5.	B8.	B9.	Ao.	A2.	A3.	A5.	Fo.	F2.	F3.	F5.	F8.
−3·9																	
3·6		1								1							
3·3																	
3·0				1													1
2·7					1												
2·4			3		1		1									1	
2·1		1			1	1			1								
1·8				1	1				1				1			1	
1·5		2	1	1		3			1	1				1			
1·2					4	1	2		1	2		1					
·9			1		1	1			2	1		1	1			2	1
·6						1	2		4			2					
− ·3		1			1				2				1			1	1
0		1					2		2			1	1				
+ ·3			1	1					2				1	1		1	
·6	1				1		1		4			1	1			1	
·9		1	1				1	1	2	4	1		1			5	
1·2					1	1		1	2	2	2	2		1		3	1
1·5									1	4						3	1
1·8					1				2	1	1	1	2			2	1
2·1									1			1	3		1	4	3
2·4												2	2	1		4	2
2·7									2			2		1		1	3
3·0			1					1				1	1	1		4	2
3·3																4	5
3·6								1				1		2		6	7
3·9									1				1			4	
4·2																2	2
4·5											1					3	4
4·8																	5
5·1																	6
5·4													1				
5·7										1						1	1
6·0																	
6·3																	
6·6																	
6·9																	
7·2																	
7·5																	
7·8																	
8·1																	
8·4																	
8·7																	
9·0																	
9·3																	
9·6																	
9·9																	
10·2																	
10·5																	
10·8																	
11·1																	

IV.

Mb, 11·4; K5, 11·7; Mb, 13·2.

	F9.	Go.	G2.	G5.	G8.	Ko.	K2.	K3.	K4.	K5.	K6.	K7.	K8.	Ma.	Mb.	Mc.	Md.
−3·9						1											
3·6																	
3·3														1			
3·0							1										
2·7										1							
2·4		1				1									2		
2·1		1		1										1			
1·8				1		1	1										
1·5						3	1			1			1	1			
1·2		2		1		8								1			
·9		1		1	1	3				2				2	1		
·6		2		1	1	7				3				1			
− ·3		1		2		4	1			2				2			
0		2		1		5								3			
+ ·3		2		1		8											
·6		1				4								1			
·9				3		3								2		1	
1·2		3		2		7	1			1				3			
1·5		1	1	1		2											
1·8		1		2		6				1							1
2·1		2		2		7	1			2							
2·4		2		5		2								1			
2·7		3		2		4											
3·0		5		1													
3·3		4		5		1											
3·6		3		3		5			1								
3·9		6		5		3				1							
4·2		6		2		2											
4·5		5		6		1											
4·8		6		7													
5·1	1	2		4		4				1							
5·4		2		4		12	2										
5·7		4		5		3	1										
6·0				5		8		1									
6·3				1		6	3										
6·6		1		2		6	1			1							
6·9				2		3				3							
7·2				1			1				1						
7·5						1	1			1							
7·8								1		1	1						
8·1						1											
8·4										1		1					
8·7						1	2			3				1			
9·0														1			
9·3										2							
9·6																	
9·9							1										
10·2														1			
10·5										1							
10·8															1		
11·1										1							1

5. *The Main Series and the Probable Error of its Spread.*—The question has often been asked as to how much of the spread of the main series is due to observational errors. Inasmuch as the above method utilises the negative parallaxes and gives a somewhat better grouping of the giants and of the dwarfs, it has been thought advisable to use this data for the purpose of attempting to answer the question. The method given herewith has been suggested to the writer by Professor Eddington.

To utilize all available data, stars having insufficient spectral classification were taken with the o of the letter. (That is, the A-type stars were grouped with Ao.) If few stars were available for certain spectral types, they were grouped with stars of adjacent spectral type.

The division between giants and dwarfs for each spectral type was first obtained from Table IV., assuming the beginning of the separation with A5, and then the stars at least one absolute magnitude above and below this division were examined. Usually the spectroscopic parallaxes of the stars within this zone made them definitely either giants or dwarfs. There were few changes from giants to dwarfs and usually not more than four stars remained in this zone.

To determine the central point either of a giant or a dwarf group of each spectral class, some mean value of absolute magnitude was assumed for the group, and from it and the visual magnitude the parallax of each star was computed. The computed parallax minus the observed absolute parallax gives a residual (v) for each star. The central point for the group adopted was the one that made as far as possible:

(First) The greatest number of small positive residuals equal to the number of small negative residuals.

(Second) The arithmetic sum of the residuals a minimum.

(Third) The sum of the positive residuals approximately equal to the sum of the negative residuals.

Such procedure does not affect the value of the centre of a group if the assumed division of giants and dwarfs is not exactly correct.

6. To obtain the probable error of the spread of each spectral class let M_c represent the value of the absolute magnitude obtained as suggested, and let π_c represent the computed parallax of a star. If π represents the observed absolute parallax of any star, then from the equation $M - m = 5 + 5 \log_{10} \pi$ we obtain for each star

$$\tfrac{1}{5}(M - M_c) = \log_{10} \frac{\pi}{\pi_c}.$$

If r_1 represents the probable error due to parallax observations, r_2 the probable error of the spread, then the combined probable error is given by the formula

$$r = 0{\cdot}845 \frac{\Sigma \left| \log_{10} \frac{\pi}{\pi_c} \right|}{n} \quad . \quad . \quad . \quad . \quad (1)$$

and from $r^2 = r_1^2 + r_2^2$, the probable error of the spread can be obtained.

The value of r from equation (1) is expressed as a logarithm, hence r_1 must be changed into a logarithm by means of the average parallax. For this reason the individual parallaxes must differ but little from the average, hence the stars under consideration must be divided into groups according to the magnitude of the computed parallax.

When the average parallax is very small the ratio $\frac{\pi}{\pi_c}$ becomes very uncertain and sometimes negative, so that equation (1) does not yield accurate results. In such a case we may use

$$r = 0{\cdot}845\frac{\Sigma|v|}{n} \quad . \quad . \quad . \quad . \quad (2)$$

which is quite satisfactory, But in this case also, the range of the individual parallaxes must not be great. This is equivalent to the statement that the stars which are treated as a group have approximately the same visual magnitude.

7. The case of the dwarf Ko stars is given in full. The separation of the giants and dwarfs was made at absolute magnitude 3·7. All stars between absolute magnitude 2·4 and 5·1 were examined, and by means of the spectroscopic parallaxes available and the magnitude of the residuals (v) the following disposition of these stars was made. Two stars with absolute magnitude less than 3·7 were taken as dwarfs, one star with absolute magnitude greater than 3·7 was taken as a giant and six stars remained in the zone. The rest were found to be distinctly either giants or dwarfs.

The absolute magnitude of 6·0 was adopted for the group as this fits best the conditions of paragraph 5. The first column of Table V. gives the McCormick catalogue star number, column two the visual magnitude, column three the computed parallax, and column four gives the absolute observed parallax. All parallaxes are given in units of 0″·001. The stars in the Table were arranged according to the magnitude of the computed parallax, and were divided into three groups, according to their computed parallax, having in mind to have as far as possible the same number of positive and negative residuals in each group.

From the first group of 24 stars we obtain :

Average parallax = 126·0, hence the corresponding value of r_1(= ±9·6) in logarithm is ±·033. From equation (1) r = ±·101, hence r_2(in logarithm) = ±·095 or r_2 (in magnitude) = ±·48.

From the second group of 14 stars we have :

Average parallax = 55·7, r_1 (in logarithm) = ±·076, r = ±·128 and r_2 (in magnitude) = ±·51.

Using equation (2) for this group, we have :

r = ±15·6, and knowing r_1 we obtain r_2 = ±12·3. This is changed

into logarithm by the help of the average parallax, hence r_2 (in magnitude) $= \pm 0.49$.

From the 21 stars of the last group and by means of equation (2) we obtain $r_2 = \pm 0.62$.

TABLE V.

Group I.

No.	m.	π_c.	π.	$\log \frac{\pi}{\pi_c}$.	v.
16	3·6	302	319	+·024	
29	3·8	276	307	+·046	
162	4·1	240	176	−·134	
618	4·8	174	184	+·024	
184	4·8	174	184	+·024	
121	4·8	174	134	−·114	
457	5·5	126	111	−·055	
286	5·7	115	112	−·011	
631	5·7	115	45	−·408	
2	5·7	115	93	−·092	
638	5·7	115	151	+·118	
133	5·8	110	176	+·204	
22	5·9	105	141	+·128	
558	5·9	105	53	−·297	
526	6·0	100	85	−·071	
95	6·1	97	80	−·084	
4	6·1	97	105	+·035	
483	6·2	92	46	−·301	
567	6·3	88	68	−·112	
584	6·4	84	110	+·117	
49	6·5	79	127	+·207	
258	6·6	76	91	+·079	
503	6·6	76	69	−·042	
176	6·6	76	54	−·148	

Group II.

616	6·7	72	29	−·395	+43
177	6·8	69	39	−·248	+30
697	6·8	69	24	−·459	+45
556	7·0	63	55	−·059	+ 8
581	7·1	60	73	+·085	−13
147	7·1	60	79	+·120	−19

Group II.—continued.

No.	m.	π_c.	π.	$\log \frac{\pi}{\pi_c}$.	v.
198	7·2	57	78	+·136	−21
452	7·3	55	54	−·008	+ 1
110	7·3	55	48	−·059	+ 7
148	7·3	55	77	+·146	−22
574	7·4	52	56	+·032	− 4
269	7·5	50	75	+·176	−25
249	7·6	48	58	+·082	−10
283	7·7	45	35	−·109	+10

Group III.

294	7·8	44	74	..	−30
484	7·9	41	35	..	+ 6
55	7·9	41	27	..	+14
132	8·0	40	8	..	+32
271	8·0	40	19	..	+21
376	8·0	40	58	..	−18
278	8·1	38	43	..	− 5
28	8·1	38	64	..	−26
5	8·1	38	45	..	− 7
72	8·2	36	10	..	+26
505	8·2	36	31	..	+ 5
441	8·2	36	49	..	−13
192	8·5	31	39	..	− 8
399	8·6	30	27	..	+ 3
384	8·6	30	67	..	−37
534	8·7	29	35	..	− 6
627	8·7	29	13	..	+16
700	8·9	27	9	..	+18
669	9·1	24	35	..	−11
492	9·1	24	72	..	−48
610	9·5	20	16	..	+ 4

There is good agreement between the different determinations; the last one naturally is the least reliable and the first the most reliable.

The weight of two was given to the first group and one to each of the other two groups.

8. Table VI. gives the summary of the results; no attempt being made to determine the probable spread in cases where too few stars were available or where the average parallax was too small.

TABLE VI.

Main Series.

Type.	No. of Stars.	Abs. Mag.	p.e.	Type.	No. of Stars.	Abs. Mag.	p.e.
B0	7			F8	43	3·8	±·54
B1	8			F9	1		
B2	4	−2·3		G0	53	4·4	±·64
B3	13			G5	58	5·2	±·71
B5	8			K0	53	6·0	±·52
B8	10	−1·5		K	6		
B9	4			K2	11	6·8	±·57
A0	30	0·3	±·80	K3	2		
A	15			K5	14		
A2	19			K6	2	8·0	±·71
A3	14	1·5	±·82	K9	1		
A5	1			Ma	3		
F0	13			Mb	3	10·3	
F2	5	2·6	±·60	Mc	0		
F	11			Md	1		
F5	48	2·9	±·67				

Giant Series.

G0	18	−0·7		Ma	19	−0·6	±1·5
G	7			Mb	3		
G5	22	−0·5		Mc	1		
G8	2			Md	1		
K0	75	0·6	±1·0				
K	10						

It appears from fig. 4 that the dwarf Ko stars are somewhat above the line of the main series and that the giant Ko stars are much below the line of the giants. The probable spread of dwarfs is much less than that of the giants, as perhaps should be expected.

It might be assumed that the lower part of the main series has a probable spread of ±·5 (this is somewhat less than the average), as the grouping of spectral types has the tendency to increase the probable spread. At the upper end of the main series the probable spread is

greater; this might be real or might be due to the possible inclusion of stars from the giant group.

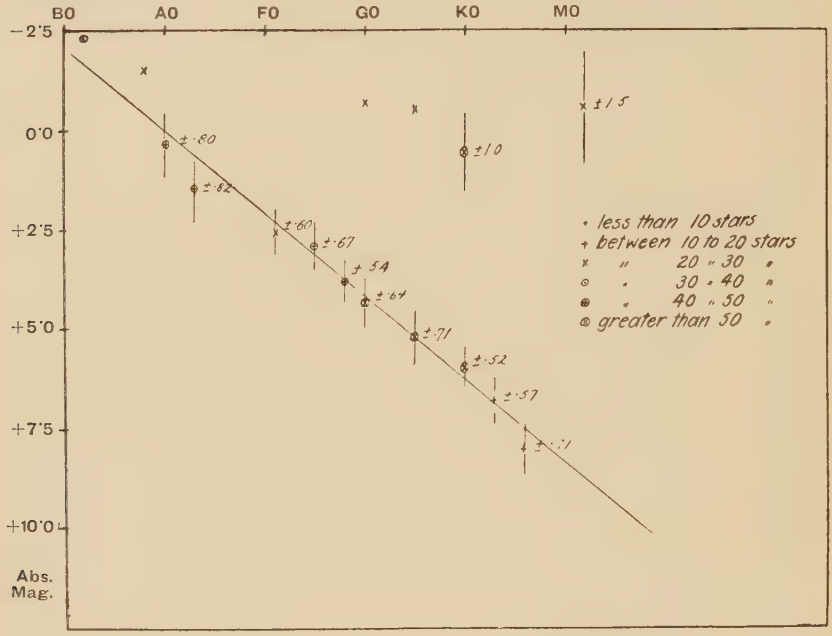


FIG. 4.

The writer is under obligation to Professor Eddington who suggested this investigation, and has given considerable advice during its progress.

Cambridge:
1928 *May.*

On the Secular Parallaxes of Faint Stars. By Jan Schilt.

(*Communicated by Professor Frank Schlesinger.*)

A study of the proper motions in certain zone catalogues has led me to the conclusion that the mean parallactic motion of stars does not invariably decrease with decreasing brightness.* This phenomenon is due to the different proportion of giants and main-sequence stars (dwarfs) for different magnitudes, and is consequently restricted to spectral classes G and later. For the earlier spectral types, in which we do not have giants, except for a few *c* stars, which are so rare as to be of no consequence in the discussion of catalogue proper motions, the decrease of parallactic motion does not reveal anything unexpected.

* *Publications of the Astronomical Society of the Pacific*, **39**, 203, 1927.

Also the Go stars behave quite normally in this respect and I have shown that these stars are practically all main-sequence stars,* at least for magnitudes greater than seven.

The G5 stars, on the other hand, show a normal decrease in proper motion up to a certain magnitude, beyond which there appears to be a well-marked increase in the mean parallactic motion.

The magnitude at which the reversal in the run of the parallactic motions occurs corresponds to the distance at which the giants begin to become rare. The fainter stars are practically all main-sequence stars. For regions in the Milky Way this phenomenon does not show if we take all spectra together; it is, however, to be expected that it will show in some degree for stars near the galactic pole, since the A and early F stars, which are the ones most likely to mask the reversal, do not occur frequently in high galactic latitudes beyond the tenth magnitude. In a recent paper, V. R. Ölander † has published photographic proper motions for 566 stars mostly between photographic magnitudes 8 and 14, in the region of Coma Berenices. These proper motions have been derived as relative ones, but they have been reduced to absolute values from a comparison with the proper motions of Boss's P.G.C. and with those of the Greenwich Catalogue for 1900.

I have collected these proper motions according to magnitudes. Median values of the components $\mu_\alpha \cos\delta$ and μ_δ and of the total proper motions μ have been obtained, as reproduced in the table.

	Median Values. (Unit ″·001.)				
Pg. Magn.	$\mu_\alpha \cos\delta$.	μ_δ.	μ.	p.	No.
7·6	−17·5	− 7·5	26	19·0	34
10·1	−10	−19	32	17·1	94
11·5	− 5	−15	30	10·9	94
12·5	− 9	−15	33	14·5	177
13·5	− 9·5	− 5·5	28·5	11·0	132
14·2	− 3	−12	27	7·8	35

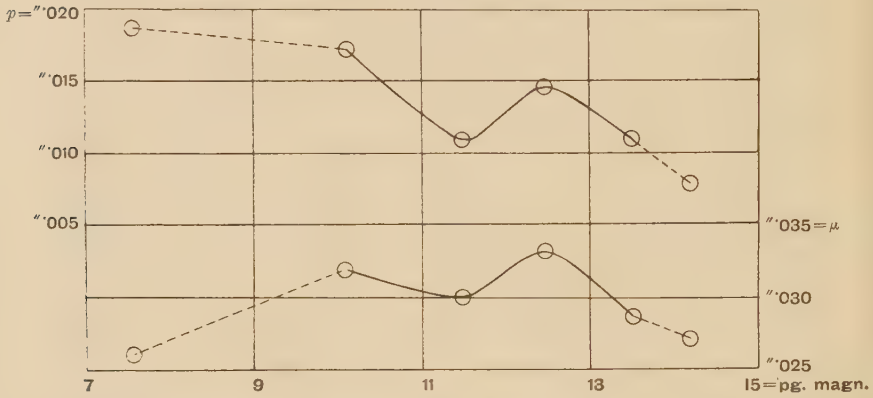
The advantage of utilising *median* values instead of the arithmetical mean is that by this procedure one gets rid of the effect of the exceptionally large proper motions in an unbiassed way. In the fifth column of the table, I have entered the parallactic motion, which has been computed from the median values of the components $\mu_\alpha \cos\delta$ and μ_δ. on the assumption that the apex is at 270° right ascension and at +30° declination. The values in the fourth and fifth columns are also represented in the figure. The two extreme points in the figure, relating to the very faintest and to the brightest stars are uncertain as a consequence of the comparatively small number of stars involved. The other points are well defined. Both curves show the predicted reversal.

It is thus shown that it is not legitimate to assume that the proper

* *Astronomical Journal*, **38**, 34, 1927.

† "Die Eigenbewegungsverhältnisse des Sternhaufens in der Coma Berenices."

motions of stars of the twelfth magnitude are negligible or small as compared with those of stars of the eleventh or tenth magnitudes. If we assume that mean parallactic motions are representative of mean parallaxes, it is further shown that a determination of the relative parallax of a star of magnitude 11·5 should as a rule yield a negative value if the comparison stars are one magnitude fainter.



The individual values of $\mu_a \cos \delta$ and μ_δ indicate a systematic motion for stars of magnitude 11·5 and in lesser a degree for those of magnitudes 10·1 and 12·5. The larger part of this peculiarity should probably be explained as the systematic motion of the giant stars with respect to the stars of the main sequence, the apex for the giants being at a considerably higher declination.*

It should, however, be mentioned that the last group (containing only thirty-five stars) of the table does not seem to follow this law.

Yale University Observatory :
1928 *April* 2.

The Orbit of Comet 1927 *IX* (*Skjellerup*).

By A. C. D. Crommelin, D.Sc.

It was noted in the last Annual Report, p. 295, that the orbit of this comet needed further correction, most of the observations being derived from circle readings, and considerably less precise than is usually the case when comparison stars can be used. It is now possible to derive accurate elements with the aid of two photographic positions obtained with the Cape astrographic telescope on December 6 and 7, and the long series of photographic positions obtained at the Cape and Johannesburg, extending from February 10 to March 31.

The first step was to obtain the fairly exact orbit given in *B.A.A. Journ.*, p. 186, using two normal places in December, and the mean of

* See reference notes above.

the Cape and Johannesburg positions for February 11. With this orbit two normal places were formed that combined (1) the Cape observations of December 6 and 7, (2) the Johannesburg ones of March 29 and 31. The following is the material on which the final orbit is based, each position being derived from two photographic plates :

U.T.	R.A. 1927·0. h m s	S. Decl. 1927·0. ° ′ ″	
1927 Dec. 6·60864	16 35 11·60	49 21 5·6	Cape plates, Dec. 6 and 7.
1928 Feb. 11·10137	18 58 28·16	27 0 33·6	Cape and Johannesburg plates, Feb. 11.
Mar. 30·07287	19 16 40·73	37 54 22·5	Johannesburg plates, Mar. 29 and 31.

The first and third places are geocentric, the middle one is for the mid-point between Cape and Johannesburg.

From these I find the following orbit for the equinox of 1927·0.

T	1927 Dec. 18·18340 U.T.
	° ′ ″
ω	47 11 13·24
Ω	77 13 29·62
ι	85 6 22·01
$\log q$	9·2462789

The Gaussian equations for 1927·0 are

$$\begin{aligned} x &= [9{\cdot}3733880]\, r \sin (v + 157^\circ\, 48'\, 21''{\cdot}42) \\ y &= [9{\cdot}9875480]\, r \sin (v + 160^\circ\, 8'\, 43''{\cdot}54) \\ z &= [9{\cdot}9999809]\, r \sin (v + 70^\circ\, 0'\, 53''{\cdot}92) \end{aligned}$$

They may also be written, the coefficients being natural numbers :

$$x = -\cdot 0771375 \tan\frac{v}{2} + \cdot 0157349\left(1 - \tan^2\frac{v}{2}\right)$$

$$y = -\cdot 3222867 \tan\frac{v}{2} + \cdot 0581886\left(1 - \tan^2\frac{v}{2}\right)$$

$$z = +\cdot 1205118 \tan\frac{v}{2} + \cdot 1656864\left(1 - \tan^2\frac{v}{2}\right).$$

The middle place is represented as follows. Observed—computed,

RA cos decl. + 1″·52, decl. + 0″·16.

These small residuals show that there is no appreciable departure from a parabola.

The following ephemeris has been formed from these elements. The light-time (which is given in units of the sixth decimal of a day) has been applied in calculating the rectangular co-ordinates of the comet, but not in those of the earth. The right ascension and declination have therefore to be interpolated for the U.T. of observation without applying light-time. I consider this to be the most convenient way of publishing cometary ephemerides.

U.T. 1927.	R.A. 1927·0 h m s	S. Decl. 1927·0. ° ′ ″	log *r*.	log Δ.	Light-Time.
Nov. 28·0	15 20 10·58	60 13 15·4	9·8342216	0·0007708	5781
28·5	15 24 34·60	59 50 52·6	9·8260539	9·9958080	5715
29·0	15 28 59·50	59 27 3·8	9·8176780	9·9907975	5649
29·5	15 33 25·04	59 1 44·3	9·8090840	9·9857417	5583
30·0	15 37 51·03	58 34 49·1	9·8002610	9·9806435	5518
30·5	15 42 17·25	58 6 13·0	9·7911972	9·9755064	5454
Dec. 1·0	15 46 43·47	57 35 50·5	9·7818806	9·9703346	5389
1·5	15 51 9·45	57 3 36·1	9·7722981	9·9651330	5325
2·0	15 55 34·94	56 29 23·4	9·7624355	9·9599076	5261
2·5	15 59 59·70	55 53 6·2	9·7522771	9·9546649	5199
3·0	16 4 23·45	55 14 38·0	9·7418070	9·9494130	5136
3·5	16 8 45·92	54 33 51·8	9·7310075	9·9441610	5074
4·0	16 13 6·84	53 50 40·3	9·7198596	9·9389195	5013
4·5	16 17 25·93	53 4 56·2	9·7083432	9·9337012	4953
5·0	16 21 42·88	52 16 31·8	9·6964363	9·9285197	4894
5·5	16 25 57·41	51 25 19·1	9·6841157	9·9233920	4837
6·0	16 30 9·21	50 31 10·3	9·6713562	9·9183367	4781
6·5	16 34 17·97	49 33 56·8	9·6581309	9·9133752	4727
7·0	16 38 23·37	48 33 30·9	9·6444109	9·9085325	4674
7·5	16 42 25·10	47 29 44·5	9·6301661	9·9038968	4625
8·0	16 46 22·81	46 22 30·0	9·6153639	9·8993201	4576
8·5	16 50 16·16	45 11 40·2	9·5999708	9·8950188	4531
9·0	16 54 4·81	43 57 8·9	9·5839522	9·8909747	4490
9·5	16 57 48·40	42 38 50·8	9·5672726	9·8872343	4450
10·0	17 1 26·55	41 16 42·5	9·5498976	9·8838499	4416
10·5	17 4 58·88	39 50 42·6	9·5317948	9·8808803	4386
11·0	17 8 24·98	38 20 52·1	9·5129362	9·8783909	4361
11·5	17 11 44·44	36 47 16·2	9·4933029	9·8764538	4342
12·0	17 14 56·81	35 10 4·5	9·4728893	9·8751474	4329
12·5	17 18 1·66	33 29 32·3	9·4517117	9·8745571	4323
13·0	17 20 58·50	31 46 1·8	9·4298183	9·8747730	4325
13·5	17 23 46·84	30 0 4·2	9·4073062	9·8758906	4336
14·0	17 26 26·20	28 12 21·1	9·3843415	9·8780031	4357
14·5	17 28 56·10	26 23 46·1	9·3611916	9·8812021	4389
15·0	17 31 16·09	24 35 26·4	9·3382321	9·8855686	4434
15·5	17 33 25·79	22 48 43·8	9·3160287	9·8911635	4491
16·0	17 35 24·99	21 5 13·7	9·2953125	9·8980171	4563
16·5	17 37 13·64	19 26 42·2	9·2770010	9·9061144	4648
17·0	17 38 51·97	17 54 58·8	9·2621333	9·9153838	4749

U.T. 1927.	R.A. 1927·0. h m s	S. Decl. 1927·0. ° ′ ″	log *r*.	log. Δ.	Light-Time.
Dec. 17·5	17 40 20·58	16 31 49·4	9·2517517	9·9256830	4863
18·0	17 41 40·38	15 18 35·9	9·2466945	9·9368133	4989
18·5	17 42 52·60	14 16 12·4	9·2474129	9·9485229	5125
19·0	17 43 58·64	13 24 52·6	9·2538408	9·9605439	5269
19·5	17 44 59·91	12 44 12·5	9·2654157	9·9726222	5418
20·0	17 45 57·68	12 13 18·3	9·2812276	9·9845444	5569
20·5	17 46 53·04	11 50 58·3	9·3002204	9·9961493	5720
21·0	17 47 46·77	11 35 54·9	9·3213773	0·0073332	5869
21·5	17 48 39·46	11 26 52·9	9·3438223	0·0180370	6015
22·0	17 49 31·51	11 22 44·7	9·3668656	0·0282367	6158
22·5	17 50 23·18	11 22 32·3	9·3900005	0·0379342	6297
23·0	17 51 14·61	11 25 27·7	9·4128687	0·0471442	6432
23·5	17 52 5·89	11 30 54·5	9·4352570	0·0558941	6563
24·0	17 52 57·10	11 38 12·6	9·4569558	0·0642040	6690
24·5	17 53 48·21	11 47 5·1	9·4779447	0·0721125	6813
25·0	17 54 39·24	11 57 9·0	9·4981629	0·0796457	6932
25·5	17 55 30·18	12 8 8·2	9·5176007	0·0868321	7048
26·0	17 56 21·01	12 19 49·9	9·5362677	0·0936977	7160

Some spectroscopic measures were made of the radial motion of the comet. The recession from the earth in kilometres per second may be deduced by multiplying the increase of light-time in half a day (expressed in units of the sixth decimal) by 0·5996, or with sufficient accuracy by 0·6. Thus it reached a maximum of 90 km./sec. between December 19·5 and 20·5.

The Cape observations of December 6 and 7 are separately represented as follows :—

Observed—Tabular, R.A. $-0^{s}\cdot18$ and $+0^{s}\cdot17$; Decl. $+3''\cdot7$ and $-3''\cdot7$.

A very careful set of circle readings was obtained at the Yerkes Observatory by Professor G. van Biesbroeck; I give them here, appending the parallax corrections :

U.T.	R.A. h m s	Par. Corr. s	Decl. ° ′	Par. Corr. ″
Dec. 19·5631	17 45 11	+·43	−12 40·7	+6·8
20·5512	17 46 58	+·41	−11 49·5	+6·3
20·9492	17 47 41	−·42	−11 37·7	+6·1
21·5474	17 48 43	+·40	−11 26·8	+6·0
22·5594	17 50 24	+·36	−11 23·3	+5·8
23·5549	17 51 57	+·35	−11 31·7	+5·6
24·53244	17 53 50	+·36	−11 48·0	+5·2

I have taken the positions as being for 1927·0, though from the method used they are probably not exactly so; the values of "Observed —Tabular" are:

In R.A. $+4^s\cdot0$, $-0^s\cdot2$, $-0^s\cdot8$, $-1^s\cdot0$, $-4^s\cdot9$, $-14^s\cdot2$, $-1^s\cdot2$; in Decl. $-49''$, $-18''$, $-26''$, $-25''$, $-29''$, $+1''$, $-13''$. The mean (excluding December 23 in R.A.) is $-0^s\cdot7$, $-23''$, thus justifying Professor Van Biesbroeck's claim that his positions were accurate to about $40''$. The following are the angular distances of the comet from the sun at the time of the near approach. The R.A. differences are reduced to great circle.

Dec. 14·5, comet 81′·04 E., 193′·2 S., Dec. 15·0, 83′·28 E., 83′·0 S., Dec. 15·5, 83′·16 E., 25′·5 N., Dec. 16·0, 80′·58 E., 130′·7 N., Dec. 16·5, 75′·41 E., 230′·7 N.

It is obvious that Mr. Chidambara Aiyar's first picture (facing p. 224) has the north point to the left, not the east point as printed; his description also implies this.

I much regret that through the wrong entry of a sign, my value for the R.A. of the radiant of the possible meteors from this comet, given on p. 487 of *M.N.*, is wrong; it should be about 33°, not 122°. I am indebted to Rev. M. Davidson for kindly pointing out the error. In view of the fact that the orbit is appreciably parabolic, I regard the occurrence of a shower as unlikely. But one may occur from the Pons-Winnecke radiant in Lyra towards the end of June. A considerable shower was seen in June 1916, nine months after the comet's perihelion passage. The interval this year is only three months longer.

Probable Errors in Determinations of the Real Paths of Meteors.

By A. King.

In an enquiry as to the extent to which the computed heights, lengths of flights, etc. of fireballs and shooting stars might be relied on it seemed best to divide the real paths into three classes, according to the standing of the observations from which they were derived. These classes are:

I. Paths derived from observations by more or less practised observers.

II. Paths derived from observations by persons familiar with the stars, but having little or no acquaintance with shooting-star work. In this section are included paths dependent partly on observations by practised workers, especially where these observations were not accurate, owing usually to a poor view of the meteor.

III. Paths derived from observations by persons almost, or quite, unfamiliar with astronomical work.

Class I.

It is in results coming under this head that, naturally, the highest degree of accuracy can be looked for. Such records as were available were collected, and the real paths of the meteors represented carefully calculated. A summary of the results is given below :

No.	Date.	G.M.T.	Mags.	H_1.	Mean Deviation.	H_2.	Mean Deviation.	Length of Path.	Velocity.	Theoretical Velocity.	Radiant.	Observers.
		h m		m.	m.	m.	m.	miles.	m.p.s.	m.p.s.	° °	
1	1900 Jan. 27	11 10	1½, >1	55·7	0·0	45·0	0·8	24	14	14·4	98−10	A.S.H., A.K.
2	1902 July 27	11 28	3, 4	66	4·1	48·2	0·1	20	24	17	283+24	A.S.H., C.L.B.
3	29	11 27	2, 2↓	54·6	0·1	47·1	0·9	15	15	17	294− 7	C.L.B., A.K.
4	Aug. 10	10 8¼	3, 1½	54·5	0·3	45	2·2	27	10	12½	196+31	C.L.B., A.K.
5	1903 Apr. 19	10 6½	>2↓, 0	59	1·9	27½	1·2	61	17½	15½	192− 6	A.S.H., A.K.
6	22	10 23	1, −1	45	1·6	24·8	0·1	21	6	11¾	150+49	C.L.B., A.K.
7	22	12 32½	0, ♀	78·5	0·5	43·6	0·2	47½	38	30½	269+32	A.S.H., A.K.
8	Nov. 15	18 7	2↓, =Sirius, >1	81	0·6	60	0·9	24	44	45	150+23	W.F.D. A.S.H., A.K.
9	1922 Jan. 3	11 19	?, 2½	73	0·1	63	1·9	27	35	34	209+45	A.G.C., A.K.
10	3	11 28	1, >1	60	2·6	49	1·3	39	35	29	236+50	W.F.D., A.K.
11	July 19	11 32	3, 1	69·4	0·3	64·6	0·4	12	35	37	50+48	J.P.M.P., A.K.
12	20	11 21	>0, 2	48	0·6	42	0·5	11	25	37	34+53	J.P.M.P., A.K.
13	28	12 9½	2↓, >1	93	1·0	80·0	0·7	21½	39	40¾	35+44	J.P.M.P., A.K.
14	31	12 45½	3, 2	65	3·2	55	2·5	11	20	28	323+27	J.P.M.P., A.K.
15	Aug. 15	10 27	1, 1	69	1·3	57	0·3	12	20	18¾	292+50	J.P.M.P., M.B.
16	15	10 30¼	2, 3½	67	3·0	55	1·0	23	18	18¾	182+60	J.P.M.P., A.K.
17	17	11 10	1½, 1	69·2	0·0	55	0·7	23	28	35½	72+64	J.P.M.P., A.K.
18	1923 Nov. 3	7 50	3, 3	55·5	0·5	51·2	0·0	12½	12½	24	52+11	J.P.M.P., A.K.
19	3	12 3	1, 3	71	2·1	56	0·3	34	43	44¼	121+22	J.P.M.P., A.K.
20	4	6 10	2, 2	73	2·9	73	0·7	39	32½	43	141+36	J.P.M.P., A.K.
21	1924 July 24	11 27¼	3, 3	65	3·2	57½	1·5	11½	20	30	326+10	J.P.M.P., A.K.
22	31	13 02¼	1½, >1	69·0	0·2	59·8	0·1	13	26	38	33+54	J.P.M.P., A.K.
23	1927 Jan. 3	13 12	0½, 2↓	68	1·2	60	2·2	21½	36	39	156− 9	J.P.M.P., A.K.
24	3	14 33	−2, 1	75	1·0	49	0·8	44	21	29	232+50½	J.P.M.P., A.K.

In the above table the G.M.T. is given in the older style, reckoning from noon to noon. The magnitudes of any given meteor are arranged as noted by the corresponding observers. H_1 and H_2 are the computed heights for the beginning and end of the flight respectively, followed in each case by the mean deviation, or average difference between the heights computed separately for each station and the adopted value. The theoretical velocity assumes parabolic speed for the meteor, with an addition due to the earth's attraction. The radiants are as yielded by

the observations ; in other words, the corrections for zenith-attraction have not been applied. The initials of observers stand for :

M.B.	M. Brockman, Ealing, London, W.
C.L.B.	C. L. Brook, Meltham, near Huddersfield.
A.G.C.	A. Grace Cook, Stowmarket, Suffolk.
W.F.D.	W. F. Denning, Bristol.
A.S.H.	A. S. Herschel, Slough.
A.K.	A. King, Leicester (Nos. 1, 3–7); Sheffield (No. 8); Ashby (following paths).
J.P.M.P.	J. P. M. Prentice, Stowmarket.

Two paths call for comment :

No. 6. The original record of the observation at Leicester has : "Diminished to 4th mag. at about 8° from end." The end-point recorded at Meltham was found to agree exactly in azimuth with the end of the bright part of the Leicester mapped track, the mean deviation of the computed heights of this point being <1 m. The flight lay much nearer Leicester than Meltham, the bright part of the path being noted 2 mags. duller at the latter place than at the former. It was clear that the final dull portion was not seen by the more distant observer, whose recorded track was therefore extended to agree with the other mapped path.

No. 8. Calculation showed that the path recorded at Sheffield, while agreeing excellently at the end with the other two, fell short at the beginning. The writer, at Sheffield, was at a great distance (much over 100 miles) from the base-line of the meteor. It was found that a backward extension of $1\frac{1}{2}°$ in the Sheffield track brought the observation into good agreement with those of Prof. Herschel and Mr. Denning, both of whom were fairly near the base-line.

The alteration of recorded paths similar to that indicated above is believed by the writer to be essential in such cases. Otherwise it is obvious that incorrect results would be obtained. It is to be noted that alteration is made only when clearly indicated by independent observations as being needed. All the other real paths in the list were calculated from the observations as they stood.

Leaving out H_2 for No. 6 and H_1 for No. 8 the average error of the beginning-points is 1·4 m., or a shade over 2 per cent., while that of the end-points is 0·9 m., or 1·7 per cent. These errors are naturally smaller for the end-points, since the finish of a meteor is usually better observed than the start.

In a similar investigation by Dr. C. P. Olivier * the average deviations for 9 meteors are 2·1 m. and 1·4 m.

With regard to the computed velocities, it is not to be expected that these will reach the status of the heights in reliability. Yet a glance down the column shows that they do not deserve to be discarded as worthless. The fact that, in the main, they are less than the theoretical velocities would seem to be due, among other things, to the retarding

* "Calculations of the Heights of Meteors observed by H. L. Alden and C. P. Olivier," *Popular Astronomy*, December 1924.

influence of the atmosphere. A similar result was found at Yale in 1899, where an occulting-device in conjunction with photography was used, yielding more accurate determinations of the duration.

Another way of assessing the reliability of these real paths is by the comparison of the results arrived at by different computers, working independently and using more or less different methods.

One detailed example will suffice, that of the Lyrid of 1903 April 22 (No. 7 of above list) :

Began.	Ended.	Length.	Velocity.	Radiant.	Computer.
m.	m.	m.	m.p.s.	α. δ.	
78	43	49	38	269°+32°	W.F.D.
$78\frac{1}{2}$	$42\frac{1}{2}$	48	35	269°+$31\frac{1}{2}$°	A.S.H.
78·5	43·6	$47\frac{1}{2}$	38	269°+32°	A.K.

Comparing the figures obtained by Prof. Herschel for the first eight meteors of the list, the averages are :

	H_1.	H_2.	Length.	Velocity.
	m.	m.	m.	m.p.s.
A.S.H.	62·9	43·2	30·2	20·4
A.K.	61·8	42·7	29·9	21·1

while a similar comparison for 19 of the paths with Mr. Denning's computations yields :

	H_1.	H_2.	Length.	Velocity.
	m.	m.	m.	m.p.s.
W.F.D.	65·3	50·4	28·0	25·4
A.K.	65·3	50·7	27·2	24·3

Class II.

Here such a degree of accuracy as in Class I. cannot reasonably be expected. The accurate mapping of meteor-tracks is acquired only by a deal of experience. It seems best, in this section, to compare the results of computers working individually and probably a typical example will be enough to give an idea of the amount of exactitude to be looked for. The fireball of 1922 February 17, $11^h\ 32^m$ is selected because it was seen by several amateur astronomers besides two regular observers (J.P.M.P. and A.K.). Both of the latter had poor views of the object, which shot over Hampshire. Comparison of real-path results :

Began.	Ended.	Length.	Velocity.	Radiant.	Computer.
m.	m.	m.	m.p.s.	α. δ.	
64	29	41	14	125°+13°	W.F.D.
65	28	48	11	119°+14°	A.K.

Class III.

Observations coming under this head are usually the most difficult to deal with, on account of their rough nature. But by experience on the part of the computer the inaccuracies of the observers can be to a large

extent allowed for. Furthermore, where the observations of a given fireball are numerous, these errors have a way of cancelling out, so to speak. We should therefore expect the results of different computers to be most divergent where the observations are few, nearest where they are many. As an example of the latter kind I take the great Yorkshire fireball of 1926 September 6, $20^h\ 43^m$ (new reckoning):

Began.	Ended.	Length.	Velocity.	Radiant.	No. of Obs.	Computer.
m.	m.	m.	m.p.s.	α. δ.		
60	28	133	15	$62°+38°$	273	W.F.D.
58	26	129	20	$60°+36°$	150–200	A.K.

In conclusion, the errors in real-path calculations seem smaller than might have been expected, especially in Class I., and it appears likely that with practised observers, well placed for the observation of a given shower and paying great attention to the careful mapping of the beginning-points and end-points of the shooting stars seen, a high degree of accuracy could be attained.

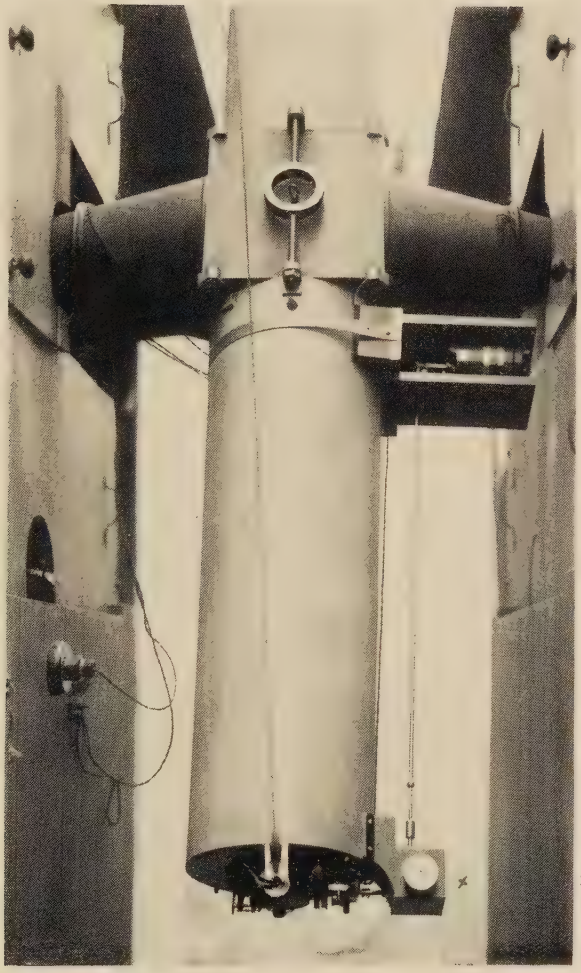
Ashby, Scunthorpe, Lincs:
1928 *May* 10.

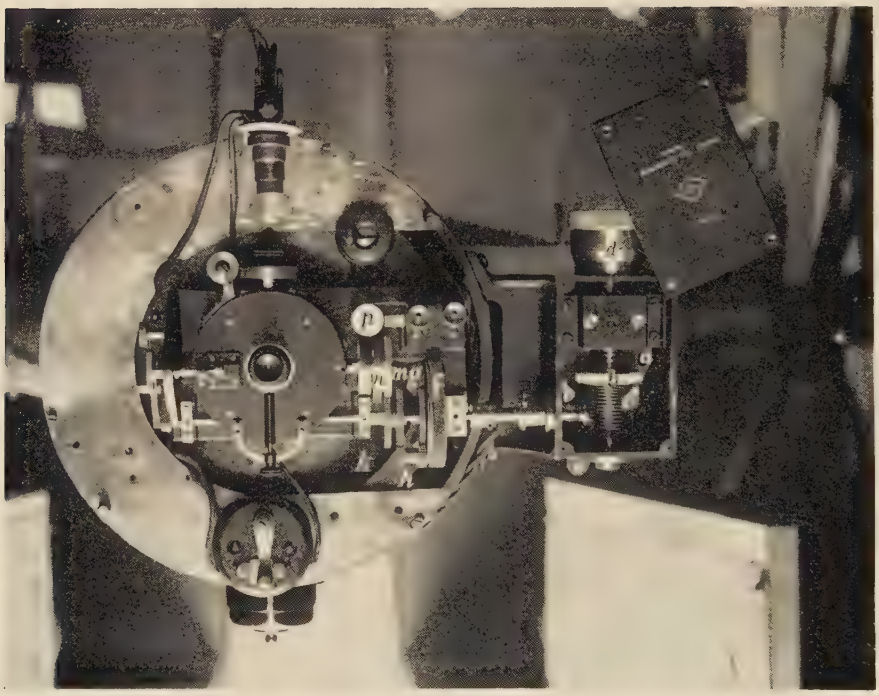
The Motor-Micrometer of the Glasgow Transit-Circle.

By Professor L. Becker. (Plates 10, 11.)

The ordinary Impersonal Micrometer is not easily manipulated. The observer has to use both hands alternately to turn a screw uniformly with a period of a few seconds. It is an art to do this and the operation is tiring to the observer. I have, therefore, designed a micrometer whose wire-frame is propelled by an electric motor at a rate somewhat in excess of that of the star's image. The characteristic feature of the design is the *retarding* of this motion by means of a planet-wheel gearing which the observer puts into operation by stopping the disc on which these wheels are mounted.

The Cambridge Instrument Company has carried out these ideas and produced a most serviceable instrument. I am especially indebted to them for the ingenious device of the gear-box which permits an alteration of the period of rotation of the micrometer screw according to the declination of the object. The arrangement of the apparatus is seen in the two photographs. Both motor and gear-box are fixed to the casing which surrounds the telescope tube of the Glasgow Transit-Circle. The motor, of $\frac{1}{8}$th horse-power, is wound for alternating current, the current being taken from the city supply. It runs 280 revolutions per second without appreciable variation. When the telescope is directed to its collimator the running of the motor produces no visible effect in the field of view. The motion of the motor is geared down to 1/300 by means of a wormwheel of 50 teeth and wheel with pinion.


← Motor.
× ← Gear-box.


p
m
mg

The connecting shaft, visible in the first photograph, has a period of 1·1 second.

The essential part of the gear-box is a circular disc (*a*) (black in the second photograph) against which a friction-wheel (*b*) (white in the photograph) is pressed, whose axis lies parallel to a diameter of the disc. The position of the friction-wheel may be altered along a radius with the effect that its period changes according to its distance from the centre of the disc. Thus by turning the milled head (*d*), speeds for all declinations, 0° to 60° and perhaps more, may be obtained. The connecting shaft turns two wheels (*c*) (*c*), gearing into each other, either of which may be put into gear with the disc for direct or reversed rotation.

Two bevelled wheels communicate the rotation of the friction-wheel to the micrometer, the first wheel (*g*) rotating at the same period as the friction-wheel (*b*). Wheel (*g*) is connected with the driving wheel (*k*) of the micrometer through the planet-wheels (*h*), whose axle is fixed on the finger-disc (*f*). The finger-disc turns free on the axle of wheel (*g*) and, if left alone, turns together with wheel (*g*). On the other hand, if it is arrested the period of wheel (*k*) is increased. The number of the teeth of the planet-wheels is such that stars of declination 0° to 30° may be observed at the same position of the friction-wheel.

The remaining wheels are arranged in the same manner as in the ordinary Impersonal Micrometer. The driving-wheel (*k*) actuates the eye-piece slide through the wheels visible at the left side of the eye-piece, and the micrometer-screw through wheel (*m*). The contact wheel is visible at (*n*). The revolution counter (*p*) is insulated, but 50° of its circumference is in metallic connection with the contact-spring of the contact-wheel (*n*). Thus signals are registered on the chronograph only during $7^s \sec\delta$ when the contact-spring of the counter touches it within the 50° mentioned. During these $7^s \sec\delta$ an electric bell is ringing. The motion of the eye-piece slide is limited by two stops, so that all the wheels are stopped when the motor has been kept running inadvertently and no harm can be done to the micrometer-screw. The connecting wheel of the eye-piece slide can be turned out of gear and the eye-piece be moved by hand. For the determination of collimation and inclination the wire-frame is moved by turning the finger-wheel (*f*).

The procedure for registering a transit is as follows. The middle wire stands at a certain reading, say 18, of the revolution counter, which is about $12^s \sec\delta$ from the position where the registering begins. The motor is switched on when the star appears, and when it is about 1^s from the middle wire (all the wires are fixed on the moveable wire-frame), the micrometer is put into gear by turning the lever (*e*) of the gear-box. Almost immediately the wire begins to move at the speed belonging to the setting on the drum (*d*) of the gear-box, and is slightly in advance of the star. During the $12^s \sec\delta$ the observer has ample time to bring the wire into coincidence with the star by placing for an instant a finger on the finger-wheel (*f*), and repeating this retarding operation when necessary. The ringing of the bell informs the observer

that the registering has begun. When it stops, the chronograph is switched off, the lever (*e*) of the gear-box is reversed and the wire-frame is moved back to the reading at starting, 18, ready for the next star, within a minute after starting. There are two break-contacts on the contact-wheel, giving 10 markings on the chronograph in 7 seconds for an equatorial star.

On the first night of trial, 1928 March 2, the writer obtained a greater accuracy than he could get previously with other modes of observation.

Statistics are enclosed in the Table, no observation being excluded apart from the first two transits. In most cases 10 records were registered. The Table contains in the second column the average departure of one record, reduced to zero position of wire, from their means. The third column gives the maximum difference between *two* such reduced records, and the fourth column shows the number of changes of sign, 0·3 expressing that the departures from the mean change sign three times out of ten possible changes. The values in this last column seem to indicate that the contacts follow too rapidly and that one contact per revolution ($1^s{\cdot}5 \sec\delta$) would be sufficient.

TABLE.

1928 March 2, Decl.	Average Error.	Difference of Extremes.	Change of Sign.
°	s	s	
+17	±0·026	0·13	0·9
+22	±0·023	0·07	0·3
+28	±0·033	0·13	0·5
+ 8	±0·057	0·33	0·5
− 4	±0·021	0·07	0·5
+ 5	±0·033	0·11	0·7
− 9	±0·046	0·21	0·5
+ 9	±0·054	0·18	0·2
+43	±0·033	0·17	0·4
− 4	±0·027	0·15	0·6
+21	±0·040	0·22	0·7
+22	±0·020	0·08	0·2
+18	±0·023	0·09	0·2
+ 7	±0·038	0·15	0·3
+11	±0·024	0·08	0·9
+22	±0·045	0·16	0·8
+ 3	±0·037	0·12	0·4
Means	±0·034	0·14	0·5

Observations of SS Aurigæ (060547, *Ch* 2195).

By P. M. Ryves.

In a previous paper * some results were given of the observations of *SS Aurigæ* made by the writer in the years 1910–1914, only the positive values, obtained during the outbursts of luminosity, being printed.

The present communication refers to the lustrum 1923–1927 and contains 503 observations, positive and negative, made on 495 nights. The method of observation has remained unaltered and, except on certain dates mentioned in the notes, the instruments employed were the same 3-inch and 5-inch refractors, generally the latter. Seven observations in 1924 (J.D.D. 2424023–58) were made at Madrid (Observatorio astronómico); six in 1925 and five in 1926 (J.D.D. 2424360–415, 769–778) at Headley, England (Rev. T. E. R. Phillips' Observatory). All the rest have been made here; position of observatory, Long. W, $0^h\ 3^m\ 14^s$, Lat. N, 41° 38′, alt. above m.s.l. 240^m (670 ft.). At certain times when there seems little probability of an outburst taking place, observations are not attempted.

In the table the calendar date is astronomical and the Julian day and fraction express Greenwich Astronomical Time (not civil). The designations of the comparison stars are those of Harvard; the letter N indicates that the variable was invisible and certainly fainter than the magnitude corresponding to the position given to N in the sequence. The positions of the comparison stars and the revised magnitudes used in reducing the estimates will be found in Table I. of the previous paper; the latter depend upon photometer measures and direct estimates made at the Harvard and Leander McCormick observatories.† For *r*, however, the value 12·3 has been used instead of 12·37, and even this is probably too faint, as the star appears considerably brighter than *q* (12·46). Star *k* is probably about 10·9 but I have used the reputed value (10·82), and *o* I continue to suspect of slight variability. The value of one grade is assumed to be 0·05 mag. and the reductions are strictly by the Argelander step method. In column W is given the weight of each observation on a scale of 1 to 5. A weight-number printed in italics means that the 3-inch telescope was used, otherwise the 5-inch is assumed except where an asterisk refers to the notes. The symbols in the last column, explained below, define the sky conditions about the variable.

Explanation of Symbols and Notes.

! = Clear transparent sky.	i = Clouds interfering.	* = 9-inch reflector.
: = Clear but less transparent.	t = Twilight.	† = 12-inch ,,
. = Clear but star low.	(= Moonlight.	‡ = 16-inch refractor.
; = Haze.	j = Unsteady.	§ = 18-inch reflector.

* *M.N.*, **84**, 8.
† See *A.J.*, 779.

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1923 Aug.					
17	3649·64	*p*10N	<12·3	3	!
18	3650·62	*q*0N	<12·5	4	!
20	3652·62	*s*4N	<13·2	4	!
26	3658·65	*p*0N	<11·8	4	(!
Sept.					
6	3669·61	*s*0N	<13·0	3	:
7	3670·59	*t*0N	<13·2	4	!
8	3671·63	*t*4N	<13·4	3	!
9	3672·60	*p*0N	<11·8	*3*	!
10	3673·65	2*p*N, *o*4N	<11·7	*3*	:
14	3677·60	(*e*10), *f*3, 5*k*	**10·58**	3	!
18	3681·58	*f*2, 4*g*, 5*k*, 6*l*	**10·60**	3	!
19	3682·54	*f*7, *g*3, *k*2, 7*l*	**10·84**	4	!
20	3683·62	*g*5, *k*5, *l*7	**11·15**	3	!
21	3684·63	*g*10, *k*10, *l*10, 4*n*	**11·34**	3	!
22	3685·67	*o*12, =*p*, *r*2, 10*q*	**12·06**	3	!
24	3687·65	*p*6N	<12·1	3	(:
25	3688·54	*p*0N	<11·8	2	(;
Oct.					
5	3698·55	*t*4N	<13·4	3	!
7	3700·52	*t*4N	<13·4	3	!
10	3703·51	*u*0N	<13·8	3	!
11	3704·51	*t*0N	<13·2	3	!
16	3709·51	*s*0N	<13·0	3	:
17	3710·55	*t*0N	<13·2	3	;
18	3711·53	*k*20N	<12±	2	i
24	3717·53	*p*8N	<12·2	3	(!
25	3718·47	*o*2N	<11·6	3	(!
26	3719·50	*o*2N	<11·5	3	(!
29	3722·49	*p*10N, *r*0N	<12·3	4	(!
31	3724·48	*m*4N, *o*0N	<11·4	3	i
Nov.					
3	3727·47	*p*0N	<11·8	*3*	!
5	3729·44	*t*0N	<13·2	3	!
6	3730·46	*p*0N	<11·8	*3*	!
7	3731·50	*p*6N	<12·1	*3*	!
8	3732·51	*p*0N	<11·8	*3*	!
11	3735·43	*s*4N	<13·2	3	!
1923 Nov.					
12	3736·46	*t*4N, *r*0N	<12·1	3	!
13	3737·45	*k*8, =*l*, 6*o*	**11·14**	*3*	!
14	3738·43	*g*6, *k*4, 2*l*	**11·00**	*3*	:
15	3739·51	*g*8, *k*10, =*l*	**11·20**	4	!
16	3740·57	*l*10, 3*o*	**11·38**	3	!
17	3741·46	*r*10, =*p*, 4*r*	**11·96**	4	(!
18	3742·56	(*r*14), *q*9, 2*s*	**12·90**	3	(!
28	3752·35	*t*0N	<13·2	3	!
Dec.					
9	3763·34	*p*0N	<11·8	*3*	!
11	3965·41	*t*4N	<13·4	4	!
12	3766·45	*t*6N, 2*u*N	<13·65	3	!
13	3767·64	*c*0N	<11·5	*3*	!
14	3768·54	*o*0N	<11·5	*2*	!
15	3769·59	2*q*N, 2*r*N	<12·3	*3*	!
16	3770·70	*r*2N, 4*q*N	<12·4	*3*	!
17	3771·36	*r*0N, 2*q*N	<12·35	3	(!
18	3772·57	*m*2N	<11·3	*3*	(!
24	3778·35	*p*0N	<11·8	3	(!
25	3779·29	*s*2, 4*t*	**13·05**	3	!
26	3780·27	*p*2N	<11·9	*3*	!
27	3781·31	*p*4N	<12·0	*3*	!
28	3782·27	*p*4N	<12·0	*3*	!
29	3783·30	*p*0N	<11·8	*3*	:
31	3785·28	*s*0N	<13·0	3	!
1924 Jan.					
1	3786·42	*m*4N	<11·4	*3*	!
2	3787·30	*t*4N	<13·4	3	!
3	3788·31	*t*6N, 2*u*N	<13·6	4	!
4	3789·30	*q*4N, *r*4N	<12·6	3	!
5	3790·65	*p*4N	<12·0	*3*	!
6	3791·26	*t*2N	<13·3	3	!
7	3792·27	*t*8N, *u*2N	<13·75	4	!
8	3793·26	*o*0N	<11·5	3	:
9	3794·57	*r*2N, 2*q*N	<12·4	3	!
10	3795·26	2*p*N	<11·7	3	!
11	3796·27	*t*0N	<13·2	3	!

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1924 Jan.					
16	3801·48	*2mN*	<11·1	*2*	(:
17	3802·29	*roN, 2qN*	<12·3	3	(!
27	3812·32	*toN*	<12·2	3	!
28	3813·56	*ooN, 2pN*	<11·6	*3*	!
29	3814·55	*p4N, roN*	<12·1	*3*	!
30	3815·54	*p4N, 2rN*	<12·0	*3*	!
31	3816·52	*p6N, qoN, 4rN*	<12·2	*3*	!
Feb.					
1	3817·50	*p1N*	<11·9	*3*	!
2	3818·50	*p4N, 2qN, 2rN*	<12·2	*3*	!
3	3819·32	*toN*	<13·2	3	!
4	3820·53	*2qN, roN*	<12·3	*3*	!
5	3821·57	*2qN, roN*	<12·3	*3*	!
6	3822·60	*2qN, roN*	<12·3	*3*	!
7	3823·58	*t2N*	<13·3	3	!
10	3826·51	*poN*	<11·8	*3*	!
12	3828·57	*o2N*	<11·6	*3*	!
14	3830·62	*ooN*	<11·5	*3*	:
24	3840·44	*ooN*	<11·5	*3*	:
25	3841·42	*qoN, 2rN*	<12·3	*3*	!
26	3842·48	*2oN*	<11·4	*3*	:
27	3843·53	*m2N*	<11·3	*3*	:
29	3845·46	*g1, 2k*	**10·77**	*3*	!
Mar.					
1	3846·52	*=g, 4k*	**10·70**	*3*	!
2	3847·45	*=g, 4k*	**10·70**	*3*	!
3	3848·40	*g3, 2k*	**10·82**	*3*	!
4	3849·50	*g2, 2k*	**10·80**	2	ii
7	3852·52	*g2, 1k*	**10·82**	4	!
8	3853·47	*g6, k2*	**11·00**	4	!
9	3854·53	*m5, =n, =o*	**11·47**	3	!
11	3856·59	*q10, r10, =s*	**12·95**	2	:
27	3872·37	*t8N, uoN*	<13·7	4	!
Apr.					
3	3879·43	*t2N*	<13·3	3	!
6	3882·43	*t6, 4u*	<13·6	3	!
8	3884·41	*moN*	<11·2	*3*	:i
13	3889·33	*2rN*	<12·2	*3*	(:
1924 Apr.					
17	3893·34	*p2N*	<11·9	3	(:
18	3894·43	*ooN*	<11·5	3	(!
20	3896·35	*p2N*	<11·9	*3*	!
21	3897·34	*2tN*	<13·1	3	!
22	3898·37	*toN*	<13·2	3	!
24	3900·36	*2tN*	<13·1	3	:
25	3901·39	*t4N*	<13·4	3	!
26	3902·36	*p2N*	<11·9	*3*	!
27	3903·42	*p4N*	<12·0	*3*	!
28	3904·36	*2p*	<11·7	*3*	:
30	3906·37	*soN*	<13·0	3	:
May					
1	3907·39	*soN*	<13·0	3	!
2	3908·37	*toN*	<13·2	3	!
6	3912·42	*p4N*	<12·0	*3*	:
7	3913·45	*p4N*	<12·0	*3*	:
8	3914·37	*2oN*	<11·4	*3*	:
9	3915·37	*2sN, 2tN*	<13·0	4	!
10	3916·37	*r4N, q2N*	<12·5	3	!
11	3917·37	*2pN*	<11·7	3	(:
12	3918·36	*p4N*	<12·0	3	(!
14	3920·37	*roN*	<12·3	3	(!
20	3926·37	*r2N, qoN*	<12·4	3	:
23	3929·37	*ro*	<12·3	3	:
26	3932·38	*roN, 2qN*	<12·3	3	!
27	3933·37	*qoN*	<12·5	3	!
29	3935·37	*2pN*	<11·7	3	i
31	3937·38	*2rN, 2qN*	<12·3	3	!
June					
2	3939·37	*moN*	<11·2	3	t:
3	3940·40	*2qN, 4rN*	<12·0	3	.
6	3943·39	*p4N*	<12·0	3	.
9	3946·39	*g4N, k2N, l2N, 2mN*	<11·0	2	(.t
July					
25	3992·61	*noN*	<11·45	*3*	!
27	3994·62	*moN*	<11·2	*3*	!
30	3997·56	*4pN*	<11·6	*3*	:
31	3998·55	*2pN*	<11·7	*3*	!

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1924 Aug.					
3	4001·58	2*p*N	<11·7	*3*	:
5	4003·56	4*p*N	<11·6	*3*	:
9	4007·66	*r*oN	<12·3	3	!
25	4023·61	*r*2, 2*q*	**12·38**	*2*	!
26	4024·55	*r*oN	<12·3	*3*	:
Sept.					
8	4037·55	*p*oN	<11·8	*3*	!
10	4039·53	2*u*N, 2*w*N	<13·75	3	(!
19	4048·55	*u*4N, *w*2N	<14·0	4	(!
24	4053·57	*y*6, 2*z*	**15·10**	3	!
29	4058·47	=*y*	**14·83**	2	:
Oct.					
6	4065·51	*p*oN, *r*oN	<12·0	*3*	:
7	4066·50	2*p*N	<11·7	3	i
8	4067·49	*p*oN	<11·7	3	(!
9	4068·51	*n*oN	<11·4	3	(;
13	4072·50	*m*4N, *n*oN	<11·4	3	(!
15	4074·48	*n*4N, 2*p*N	<11·7	3	(!
16	4075·50	*p*4N	<12·0	3	(!
19	4078·50	*r*oN	<12·3	3	(!
22	4081·52	*p*4N	<12·0	*3*	!
24	4083·50	*t*oN	<13·2	3	:
29	4088·52	*t*2N	<13·3	3	!
30	4089·56	*u*oN	<13·8	3	!
31	4090·53	*t*oN	<13·2	3	!
Nov.					
1	4091·46	*w*4N	<14·1	3	!
2	4092·51	*t*4N	<13·4	4	!
3	4093·51	*p*oN	<11·8	*3*	:
13	4103·47	*n*oN	<11·4	3	(!
14	4104·49	*n*oN	<11·4	*3*	(!
16	4106·43	*r*oN	<12·3	3	(!
17	4107·42	*n*oN	<11·4	*2*	!
18	4108·49	*p*6N, 2*r*N	<12·1	*3*	!
19	4109·51	*n*oN	<11·4	*3*	!
20	4110·39	*t*oN	<13·2	3	!
21	4111·76	*p*oN	<11·8	3	(!t
22	4112·49	*u*oN	<13·8	3	!
1924 Nov.					
26	4116·52	*r*oN	<12·3	*3*	!
27	4117·40	2*r*N	<12·2	*3*	!
Dec.					
3	4123·55	*r*oN	<12·3	*3*	!
4	4124·57	*q*oN	<12·5	*3*	!
6	4126·47	*r*2N	<12·5	3	(!
7	4127·37	*n*2N	<11·5	3	(;
11	4131·41	*f*6, 4*k*	**10·72**	3	(:
12	4132·41	*f*4, 4*k*	**10·67**	3	(:
13	4133·41	*f*4, 5*k*	**10·63**	3	(
14	4134·32	*f*3, 4*k*	**10·64**	2	!
16	4136·50	*f*7, =*k*	**10·83**	2	!
17	4137·42	*k*7, 6*m*	**11·04**	4	!
1925 Jan.					
23	4174·28	*t*2N	<13·3	3	!
29	4180·28	*q*oN	<12·5	3	(:
Feb.					
10	4192·29	*t*oN	<13·2	3	!
11	4193·31	*t*4	<13·4	3	!
15	4197·32	2*p*N	<11·7	*1*	!
18	4200·42	*r*4N, *q*oN	<12·5	*4*	!
19	4201·46	*t*oN	<13·2	3	!
20	4202·57	*p*2N	<11·9	*3*	!
22	4204·30	(*p*6N), *r*oN	<12·2	*4*	!
23	4205·43	*t*4N	<13·4	4	!
25	4207·42	2*p*N	<11·7	*3*	i
27	4209·51	*r*oN	<12·3	*3*	!
28	4210·47	*p*2N	<11·9	*4*	:
Mar.					
1	4211·45	2*t*N	<13·1	4	!
5	4215·35	*p*2N	<11·9	3	(i
8	4218·39	*p*6N	<12·1	3	(!
12	4222·33	*p*6N	<12·1	*3*	!
14	4224·40	*t*oN	<13·2	3	!
16	4226·49	*p*oN	<11·8	*3*	!
17	4227·50	*p*oN	<11·8	*3*	!
18	4228·50	*p*oN	<11·8	*3*	!

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1925 Mar.					
19	4229·34	*n*6, *p*3	**11·84**	2	!
19	4229·52	*n*9, 2*p*	**11·80**	3	!
20	4230·51	*g*5, *k*5	**11·05**	*2*	:
21	4231·42	=*g*, 3*k*	**10·72**	*3*	!
22	4232·48	*g*6, *k*3	**11·02**	*3*	!
23	4233·40	=*m*	**11·2**	*1*	i
25	4235·42	4*p*	**11·58**	*1*	!
26	4236·49	=*r*	**12·3**	*1*	!
Apr.					
30	4271·36	*p*o*N*	<11·8	3	(i
May					
2	4273·42	*p*6, (*r*)	<12·1	3	(i
7	4278·36	*p*4	<12·0	4	(!
10	4281·38	*q*2*N*	<12·6	4	!
12	4283·43	*m*o*N*	<11·2	*3*	.
13	4284·41	*r*2*N*, *q*2*N*	<12·5	4	:
14	4285·37	*q*8*N*	<12·9	2	:
19	4290·42	*m*o*N*	<11·2	*3*	:
20	4291·41	*p*2*N*	<11·9	3	.
23	4294·41	*r*2*N*, *q*o*N*	<12·4	4	:
26	4297·42	2*r*N, 2*q*N	<12·25	4	.
30	4301·40	2*p*N	<11·7	3	(:
31	4302·37	*m*4*N*, 2*n*N	<11·4	3	(:
July					
28	4360·57	*w*5*N*	<14·4	4	!
Aug.					
15	4378·59	*r*2, 2*q*	**12·38**	4	;§
16	4379·60	*q*9, 6*t*	**12·90**	3	!§
17	4380·59	=*w*, (11*x*)	**13·96**	4	:§
29	4392·62	=*y*	**14·8**	1	!§
Sept.					
21	4415·52	*w*4*N*	<14·1	2	;§
Oct.					
5	4429·52	2*p*N	<11·7	3	(!
6	4430·51	*p*o*N*	<11·8	3	(!
7	4431·51	*m*o*N*	<11·2	3	(:
9	4433·51	*m*o*N*	<11·2	*2*	:
12	4436·50	*r*1o*N*, *q*o*N*	<12·6	4	!

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1925 Oct.					
13	4437·53	*t*4*N*	<13·4	3	!
15	4439·54	*t*o*N*	<13·2	3	:
16	4440·61	*t*6*N*, 2*u*N	<13·6	3	!
17	4441·49	*t*6	<13·5	3	!
18	4442·52	*t*4*N*, (*u*)	<13·4	3	!
19	4443·57	*t*4*N*	<13·4	3	!
20	4444·54	*w*4*N*, (*x*)	<14·1	4	!
21	4445·54	*u*2*N*, *w*o*N*	<13·9	3	!
22	4446·52	*n*o*N*	<11·5	*3*	i
23	4447·53	*p*4*N*, 2*r*N	<12·1	*3*	!
24	4448·44	*p*o*N*	<11·8	*3*	:
25	4449·56	*r*4*N*, *q*o*N*	<12·5	3	!
26	4450·59	*t*6*N*, (*u*)	<13·5	4	!
31	4455·56	*n*o*N*	<11·5	3	(:
Nov.					
2	4457·55	*m*4*N*, 2*n*N	<11·4	3	(;
3	4458·45	*l*4*N*	<11·2	3	(;
5	4460·58	*p*4*N*	<12·0	3	(:
8	4463·54	*p*o*N*	<11·8	*3*	!
9	4464·49	2*r*N	<12·2	*3*	!j
10	4465·59	*r*o*N*	<12·3	*3*	!
11	4466·47	*r*4*N*, 4*q*N	<12·4	*3*	!
12	4467·50	*r*o*N*	<12·3	*3*	:
13	4468·47	*r*o*N*	<13·3	*3*	!
14	4469·51	*t*4*N*, (*u*)	<13·4	4	!
16	4471·60	*p*2*N*	<11·9	*3*	!
17	4472·44	*w*4	**14·14**	2	!
18	4473·52	=*g*, *k*4	**10·90**	4	!
19	4474·63	*g*2, 4*k*	**10·75**	*2*	i
20	4475·61	*g*2, 2*k*	**10·80**	*2*	!
21	4476·44	1*g*, 1*k*	**10·75**	4	!
23	4478·43	*g*4, *k*2	**10·94**	2	(;
23	4478·52	*g*3, *k*2	**10·92**	2	:
25	4480·46	*g*3, *k*3	**10·95**	2	(!
26	4481·42	*g*8, *k*12, 4*n*, 4*m*	**11·21**	3	(;
Dec.					
14	4499·42	*u*o*N*, 2*w*N	<13·8	3	!
25	4510·67	2*s*N	<12·9	3	:

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1926 Jan.					
2	4518·29	*g*3, *k*3	**10·97**	4	!
3	4519·30	=*m*, =*n*	**11·33**	3	!
4	4520·28	*m*2, (*n*8)	**11·6**	2	(!
5	4521·55	*r*8, *q*2	**12·63**	2	(!
6	4522·33	*t*5	**13·45**	3	!
Feb.					
3	4550·45	*t*oN	<13·2	3	!
4	4551·36	*t*6N	<13·5	3	!
7	4554·44	*g*6, =*k*	**10·95**	3	!
9	4556·42	*m*8, 3*n*	**11·45**	2	!
10	4557·52	1*r*, 4*q*	**12·25**	4	!
11	4558·32	*q*9, 12*t*	**12·8**	2	!
26	4573·49	*p*oN	<11·8	3	(!
Mar.					
4	4579·42	*p*2N	<11·9	*3*	:
6	4581·51	*p*oN	<11·8	*3*	!
7	4582·44	*p*2N	<11·9	*3*	!
8	4583·48	*r*1oN, *q*6N, (*s*)	<12·8	3	!
9	4584·46	*r*6N, *q*2N	<12·6	3	!
10	4585·42	2*p*N	<11·7	*3*	!
11	4586·35	*t*oN	<13·2	3	!j
12	4587·49	*r*2N, *q*oN	<12·4	*3*	!
13	4588·47	*r*12N, *q*8N	<12·9	3	!
14	4589·40	*s*oN	<13·0	4	!j
15	4590·53	*p*2N	<11·9	*3*	!
16	4591·50	*r*oN, *q*oN	<12·4	3	!
17	4592·45	2*s*N	<12·9	3	:
18	4593·43	*r*4N, *q*oN	<12·5	3	(:
20	4595·59	*l*oN	<11·0	*2*	.
21	4596·35	*p*oN	<11·8	3	(!
22	4597·43	*p*oN	<11·8	3	(!
24	4599·49	*p*4N	<12·0	3	(;
25	4600·533	*p*oN	<11·8	3	(:
30	4605·43	*p*2	**11·9**	1	(!
31	4606·35	*r*3, =*q*	**12·45**	2	!
Apr.					
1	4607·37	*q*6N	<12·8	3	(:
2	4608·40	2*s*N	<12·9	..	:

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1926 Apr.					
22	4628·35	2*r*N	<12·2	3	(!
May					
3	4639·40	*p*4N	<12·0	3	i
4	4640·38	2*t*N	<13·1	3	!
7	4643·37	*r*4N, *q*2N	<12·5	4	:
10	4646·37	*r*4N, *q*oN	<12·5	4	!
12	4648·38	*m*6N	<11·5	2	i
21	4657·37	*g*10, *k*4, *l*2, 1o*n*	**11·10**	2	(:
22	4658·37	*g*4, =*k*, (=*l*)	**10·92**	2	(:
24	4660·37	*g*8, *h*2, 2*l*	**11·00**	2	(:
28	4664·37	*g*9, =*k*, 2*l*2	**11·08**	2	!t
29	4665·37	*m*2, 5*n*	**11·25**	2	.
30	4666·38	*p*4	**12·0**	1	.t
Aug.					
7	4735·65	*g*2, *k*1	**10·87**	3	!
8	4736·59	*g*6, *k*2	**11·00**	*2*	:
10	4738·62	=*m*	**11·2**	*1*	:
11	4739·65	*o*4, (4*p*)	**11·63**	2	!
12	4740·64	*q*6	**12·76**	3	!
Sept.					
10	4769·54	*x*oN	<14·5	3	!†
12	4771·55	1*r*, 4*q*	**12·25**	4	!*
15	4774·52	*n*2, 2*o*	**11·48**	2	;†
18	4777·57	*t*5	**13·45**	3	i†
19	4778·57	*u*2, 1*m*	**13·89**	4	!†
Oct.					
2	4791·54	*q*2N	<12·6	3	!
5	4794·49	*r*oN	<12·3	3	:
8	4797·51	*t*2N	<13·3	3	!
9	4798·51	*t*4N	<13·4	4	!
10	4799·50	*t*oN	<13·2	3	:
11	4800·58	*t*oN	<13·2	3	!
12	4801·54	*t*4N	<13·4	4	!
13	4802·50	*t*4N	<13·4	4	!
14	4803·61	*t*8N	<13·6	3	!
15	4804·52	*t*2N	<13·3	3	:
16	4805·47	*r*2N	<12·4	3	(:
17	4806·71	*s*oN	<13·0	3	!

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1926 Oct.					
18	4807·49	p_4N, (*r*)	<12·0	4	(:
18	4807·71	*s*oN, (*t*)	<13·0	3	!
22	4811·46	*n*oN	<11·45	3	(:
23	4812·48	2*n*N, *o*2N	<11·5	3	(!
24	4813·45	*n*2N, *o*4N	<11·6	3	(!
25	4814·57	n_4N, *o*2N	<11·6	3	(:
26	4815·50	*r*2N, (*q*)	<12·4	4	(:
Nov.					
1	4821·54	*q*6	**12·75**	3	!
2	4822·43	*t*6	**13·5**	1	:
25	4845·46	2*t*N	<13·1	3	!
27	4847·47	*q*2N	<12·6	2	:
29	4849·42	*u*oN, *w*oN	<13·85	3	!
Dec.					
1	4851·42	t_4N, (*u*, *w*)	<13·4	3	!j
2	4852·49	2*t*N	<13·1	3	:j
5	4855·35	*t*2N	<13·3	3	!j
6	4856·42	q_4N	<12·7	3	:j
7	4857·42	*q*6N	<12·8	3	!j
8	4858·35	*t*oN	<13·2	4	!
9	4859·43	*w*oN, (*u*oN)	<13·85	4	!
10	4860·42	2*t*N	<13·1	3	!
11	4861·47	*q*oN, (*r*3N)	<12·6	3	;
20	4870·30	*g*2, *k*1	**10·90**	2	(;
21	4871·30	=*g*, *k*2	**10·85**	2	:
23	4873·34	*g*2, *k*3	**10·92**	3	!
24	4874·30	*l*9, =*m*	**11·33**	2	!
25	4875·30	*n*5, 6*r*	**11·9**	2	;
1927 Jan.					
22	4903·55	r_5N, *q*oN	<12·5	4	(!
23	4904·29	2*t*N	<13·1	3	:
24	4905·68	*r*oN	<12·3	3	(:
25	4906·30	2*t*N	<13·1	3	!
26	4907·29	*s*oN	<13·0	3	:
27	4908·28	*t*oN	<13·2	3	!
28	4909·29	*t*oN	<13·2	3	!
30	4911·27	*t*2N	<13·3	3	!

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1927 Jan.					
31	4912·30	t_4N	<13·4	3	!
Feb.					
1	4913·29	*t*oN	<13·2	3	!
2	4914·54	*t*oN	<13·2	3	!
3	4915·30	t_4N	<13·4	3	!
4	4916·30	*t*oN	<13·2	3	!
5	4917·29	2*s*N	<12·9	3	:
6	4918·29	*m*oN	<11·2	3	(!
7	4919·56	*q*6N	<12·8	3	!
8	4920·28	*r*oN	<12·3	3	(!
8	4920·58	2*s*N	<12·9	3	!
13	4925·28	*m*oN	<11·2	3	(!
14	4926·53	*m*oN	<11·2	3	(!
15	4927·52	2*p*N	<11·7	3	(!
16	4928·49	2*p*N	<11·7	3	(;
17	4929·30	*r*2N	<12·4	4	(:
19	4931·29	*q*oN	<12·5	3	!
20	4932·29	*q*oN	<12·5	3	:
21	4933·42	2*s*N	<12·9	3	!
22	4934·43	*t*oN	<13·2	3	!
24	4936·35	*g*6	**11·07**	2	i
25	4937·30	1*g*, 4*k*	**10·67**	3	:
25	4937·47	=*g*, 4*k*	**10·70**	2	!
27	4939·34	*m*2, 3*n*	**11·30**	2	!
27	4939·50	*m*5, =*n*	**11·45**	3	!
28	4940·56	*p*2, 4*r*	**11·98**	2	!
Mar.					
1	4941·45	*q*3, *r*3	**12·53**	3	!
3	4943·42	*t*oN	<13·2	3	:
20	4960·39	*q*oN	<12·5	3	(!
27	4967·35	*t*6N, (*u*, *w*)	<13·5	3	!
28	4968·47	q_4N	<12·7	3	!
30	4970·50	*s*oN	<13·0	3	:
31	4971·37	q_4N	<12·7	3	:
Apr.					
1	4972·51	*q*oN	<12·5	3	!
2	4973·43	q_4N	<12·7	3	:
3	4974·36	*t*6N, 2*u*N	<13·6	4	!

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1927 Apr.					
4	4975·42	*s*oN	<13·0	3	!
5	4976·45	*s*oN	<13·0	3	!
6	4977·50	2*s*N	<12·9	3	:
7	4978·41	*r*oN	<12·3	3	(!
8	4979·43	*r*2N, (*q*)	<12·4	4	(!
9	4980·46	*p*6N, (2*r*N)	<12·1	3	(:
10	4981·45	*p*4N	<12·0	3	(!
11	4982·37	*n*2N	<11·55	3	(!
12	4983·43	*m*4N, *o*oN	<11·4	3	(!
13	4984·35	*n*2N	<11·55	3	(!
14	4985·38	*m*oN	<11·2	3	(:
15	4986·34	*n*oN	<11·45	3	(:
16	4987·33	*n*2N	<11·55	3	(!
17	4988·34	*r*oN	<12·3	4	!
18	4989·34	*s*4N, 2*t*N	<13·15	3	!
19	4990·37	*s*4N	<13·2	3	!
20	4991·40	*t*2N	<13·2	4	!
21	4992·39	*t*6N, 2*u*N	<13·65	3	!
22	4993·39	*s*4N, (2*t*N)	<13·2	3	!
23	4994·39	2*s*N	<12·9	3	:
25	4996·39	2*w*N, (*u*4N)	<13·9	4	!
26	4997·39	*s*oN	<13·0	3	!
27	4998·39	2*s*N	<12·9	3	i
28	4999·35	*s*oN	<13·0	3	i
29	5000·38	2*s*N	<12·9	3	:
30	5001·40	*s*2N, 2*t*N	<13·1	4	!
May					
7	5008·44	*q*4N	<12·7	3	(!
8	5009·39	*q*oN	<12·5	3	(!
12	5013·37	*p*4N	<12·0	3	(!
13	5014·38	*p*oN	<11·8	3	(!
14	5015·38	*g*4, =*k*	**10·90**	2	(:
16	5017·42	*g*2, 4*k*	**10·74**	2	(:
18	5019·39	*g*2, 3*k*	**10·77**	3	:
19	5020·36	*g*2, 2*k*	**10·80**	4	:t
21	5022·39	*g*4, =*k*	**10·90**	3	:
22	5023·39	*k*4, 4*m*	**11·14**	2	:
23	5024·40	*k*10, *m*7, 2*o*, *n*3	**11·47**	3	.j

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1927 July					
24	5086·62	*l*oN	<11·0	*3*	!
28	5090·62	*n*oN	<11·45	2	!
Aug.					
3	5096·59	*k*4N	<11·0	*3*	:
5	5098·64	*q*2N, (*r*8N)	<12·6	3	!
8	5101·61	*m*oN	<11·2	*3*	:
9	5102·64	*t*6N, 2*u*N	<13·6	3	!
20	5113·63	*g*2, *k*1	**10·87**	3	(!
22	5115·57	*m*1, 1*n*	**11·33**	*1*	i
24	5117·60	(*r*6), *q*1	**12·53**	4	!j
Sept.					
20	5144·53	*s*2N	<13·1	3	!
22	5146·61	*t*oN	<13·2	3	!
25	5149·66	*t*4N	<13·4	3	!
26	5150·53	*t*oN	<13·2	3	!
28	5152·55	*t*4N	<13·4	3	!j
30	5154·67	*t*6N	<13·5	3	!j
Oct.					
1	5155·49	*t*4N	<13·4	4	!
2	5156·54	*u*oN, *w*oN	<13·85	3	!
3	5157·52	*u*oN, *w*oN	<13·85	3	!
4	5158·54	(*u*1N), *w*oN	<13·9	3	!
5	5159·57	*w*oN	<13·9	3	!
6	5160·58	*w*oN	<13·9	3	!
7	5161·61	*t*oN	<13·2	3	!
8	5162·67	*t*oN	<13·2	3	!
10	5164·61	*p*oN	<11·8	3	(!
11	5165·59	*n*2N	<11·5	3	(!
12	5166·58	*p*4N, (*r*)	<12·0	3	(!
17	5171·50	*g*8, *k*8, =*l*, 8*n*	**11·11**	3	:
18	5172·48	*g*8, *k*10, 2*m*, 4*m*	**11·21**	2	!j
19	5173·53	*o*4, 2*p*, 8*r*	**11·76**	3	
20	5174·52	*r*2, =*q*	**12·48**	3	!
Nov.					
2	5187·56	*w*2N	<14·0	4	!
6	5191·68	*q*oN	<12·5	3	:
10	5195·55	*n*oN, *o*oN	<11·5	3	(:
13	5198·36	*r*oN	<12·3	3	!j

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1927 Nov.					
15	5200·49	*r*6N, *q*2N	<12·6	3	!
16	5201·49	*t*oN	<13·2	3	!
17	5202·35	*r*oN	<12·3	3	i
18	5203·42	*t*4N	<13·4	3	!
20	5205·47	*t*oN	<13·2	3	!
23	5208·45	*t*oN	<13·2	3	:
27	5212·37	*s*oN	<13·0	3	!
28	5213·42	*q*oN	<12·5	3	:
29	5214·49	*r*6N, *q*oN	<12·5	3	!
30	5215·47	*t*6N, 1*w*N	<13·7	3	!

	J.D. 242 . . .	Comparisons.	Mag.	W.	Sk.
1927 Dec.					
3	5218·48	*r*2N	<12·4	3	(!
4	5219·44	*r*oN	<12·3	3	(:
5	5220·47	*r*oN	<12·3	3	(!
5	5220·72	*s*oN, 2*t*N	<13·1	3	:
7	5222·48	*m*oN	<11·2	2	(;
9	5224·30	*m*oN	<11·2	2	(!
17	5232·60	2*m*, =*n*	**11·28**	2	(!
20	5235·30	=*r*	**12·3**	2	:

In 1923 (August to December only) *SS Aurigæ* was seen at or near three maxima and in the following years 3, 3, 6, and 5 maxima, respectively, were detected, the number corresponding to 1926 being the largest so far observed here in any one year.

The existence of a class of maximum intermediate between the so-called "long" and "short" types, suggested in the previous paper, receives some confirmation from the later observations. I think that in such cases the rise, especially the last part, is somewhat slower than in the ordinary long and short types (though not to be compared with the slow rise to an "abnormal" maximum), the tendency being to produce a pointed maximum. It also seems possible that the majority of the maxima do not conform strictly to typical mean curves, as has been supposed, but rather that there are small differences in the character of the variation on the various occasions included within the same general type.

Zaragoza, Spain :
1928 *April.*

Observations of Novæ, 1927–1928. By W. H. Steavenson.

The following observations were, unless otherwise stated, made with the 6-inch Wray refractor. Those marked "G" were made with the 28-inch equatorial of the Royal Observatory, Greenwich, by kind permission of the Astronomer Royal.

Nova Persei (1901).

Variability has again been marked in the period under review, the total range amounting to fully 1·5 magnitudes. Unfortunately there are many large gaps in the present series of observations. Bad weather was largely responsible for this, and not for many years have

so few estimates been obtained in one season. The observations are for the most part too scattered to allow of any useful deductions as to the possible period of the oscillations. Perhaps the points most worthy of note are the sudden drop of $0^m.6$ in 24 hours between 1927 October 17 and 18, and the more or less steady fall through nearly $1^m.5$ in the first two months of 1928. The small wisp of nebulosity attached to the nova on its south preceding side was faintly seen at Greenwich under somewhat poor conditions on 1928 January 8. It appears to have undergone but little change in shape or brightness for some years. It was first seen by the writer in 1916 September, and was detected independently by Barnard in December of the same year.

Date.	Mag.	Date.	Mag.
1927 July 21	13·72	1928 Jan. 11	12·69
26	13·7±	13	12·76
Sept. 22	13·28	15	12·69 G
24	13·18	19	12·74 G
25	13·23	22	12·96
29	13·18	24	12·54
Oct. 2	13·67 G	26	12·79
17	13·28	27	12·74
18	13·88	Feb. 17	13·23
22	13·42	21	13·42
23	13·77	26	13·67 G
Nov. 6	13·6±	Mar. 4	13·83 G
1928 Jan. 8	12·37 G	23	13·5±
9	12·43	Apr. 6	13·28 G
10	12·48	8	13·28 G

Nova 100. 1927 *Tauri.*

This object was first detected by Messrs. Schwassmann and Wachmann on plates taken at Bergedorf in 1927 November. Its position for 1927·0 is R.A. $5^h\ 15^m\ 11^s.50$, Decl. $+16°\ 38'\ 30''.3$. It precedes by 34^s and is $53''$ south of the star B.D. $+16°.755$ (mag. 8·6). Two photographs of the field of the nova will be found in *A.N.*, 5560.

Seen with the 6-inch refractor on 1927 November 24 the nova appeared to be of about the 10th magnitude. It was estimated to be $0^m.3$ brighter than a small star 2′ or 3′ *sp.*, and about the same amount fainter than B.D. $+16°.753$, which lies 6′ or 7′ to the north. There was nothing in the colour or appearance of the nova to distinguish it from other stars in the field.

Nova Aurigæ (1892).

Observed with the 28-inch telescope on 1928 January 15, 19, and 22. On all three occasions it appeared very slightly fainter than the

little star which immediately follows it. This is Burnham's "B" (see *M.N.*, **52**, 433). Adopting Burnham's estimate of $14^{m}\cdot8$ for this star I obtained the following values for the brightness of the nova :—

1928 January 15	14·85
19	14·90
22	14·85

Almost certainly it was a little fainter than this in 1926 January (see *M.N.*, **86**, 367), but the variability appears to be very slight. Fortunately, the star B is so close to the nova and so nearly of the same magnitude that very small changes of brightness can readily be detected.

Nova Geminorum (1912).

Observed at Greenwich on 1928 January 15 and 22. On both these nights it was compared with a small star 2′ *np*. Assuming the magnitude of this star to be 14·4, the nova was estimated to be of magnitude 14·7 on January 15 and 14·6 on January 22. Probably no real change took place between the two observations. The discrepancy is quite as likely to be due to the great difficulty of making accurate estimates of the brightness of this nova, on account of its close proximity to a 13th magnitude star which lies a few seconds of arc to the north. When this brighter star was occulted by a field-bar it was noted on January 15 that the nova seemed to be involved in nebulosity. I have observed this in previous years but could never be entirely certain of its not being an optical effect. At my request the Rev. T. E. Espin examined the nova on the nights of 1928 January 26 and 27 with his 24-inch Calver reflector. He reports that the conditions were excellent, but that he "could see no nebulosity about the nova, and it and the *comes* seemed equally sharp." He adds the following interesting note :—

"Using a low power (field about 45′), and sweeping rapidly backwards and forwards, it was obvious that the field of the nova is occupied by a dark nebula, very like the one found by me in Perseus (*M.N.*, **58**, 334). It is oval, the major axis being marked by B.D. + 32°·1437 and B.D. + 31°·1440. The nova is fairly central. The diameters of the major and minor axes are roughly 40′ and 20′. The darkest portion is north of the nova. It seemed a dark reddish colour."

Mr. Espin's estimate of the magnitude of the nova on January 27 was 14·5, using the comparison star already mentioned.

Nova Aquilæ (1918).

This nova was estimated to be of mag. 10·7 on 1927 September 24. It appears now to have reached a practically stationary condition. Indeed it would seem that its light is less variable now than it was before its outburst in 1918.

Nova 60. 1927 *Aquilæ.*

The position of this object (discovered by Dr. Max Wolf) is R.A. $18^h\ 52^m\ 11^s{\cdot}85$., Decl. $-3°\ 24'\ 59''{\cdot}8$ (1927·0). It lies 10′ south of the star B.D. $-3°{\cdot}4417$ (mag. 9·1). On 5 nights its light was compared with that of this star and also with that of $-3°{\cdot}4418$, which is about $0^m{\cdot}4$ fainter. The estimates are as follows:—

1927 August 27	9·1
28	9·1
31	9·3
September 1	9·3
24	9·7

Nova Cygni (1920).

This nova is still slowly fading. It was observed on one night only (1927 September 24) and was then estimated to be $0^m{\cdot}4$ brighter than the small star immediately preceding it. This would make its magnitude about 14·1.

Nova Lacertæ (1910).

Observed at Greenwich on 1928 January 22. The magnitude was estimated to be 14·3. This is about $0^m{\cdot}2$ fainter than most of the estimates of recent years. Generally, however, this nova has been observed with the 6-inch telescope, and it may be that the discrepancy merely represents some systematic error affecting observations of so faint an object with so inadequate an aperture.

Note on Seasonal Changes in the Length of the Day.
By H. C. Plummer, F.R.S.

Some time ago the mind of the writer was impressed, owing to casual circumstances, by the apparently marked closing in of the day during the last three weeks of August. This impression naturally suggested a question. At what times of year are the rates of change in the length of the day greatest? There is nothing subtle in this trivial problem, and for the practical purpose refraction and the eccentricity of the Earth's orbit can be neglected.

The answer appears obvious. At the solstices the rate of change is zero. At equal intervals before and after an equinox the relative lengths of day and night are interchanged, but the rate of change of either is the same. This symmetry with respect to an equinox is enough to show that at the corresponding time of year the rate of change in the length of the day is stationary. It is natural to conclude that the rate at that time is greatest. But the question requires a little closer examination.

If, on the other hand, the problem is approached practically by

inspecting a sunrise and sunset table for an English latitude, it is found that for nearly six weeks on either side of the two equinoxes the rate of change is almost constant. For nearly half the year the hours of daylight are gaining or losing a little less than four minutes a day. Thus the maximum in the rate is extremely flat, and the ordinary table is unsuitable for the study of possible small changes. The vague impression mentioned above is, however, fairly substantiated by the following extract :—

1923.	Sunrise.		Sunset.		Interval.	
	h m		h m		h m	
Aug. 11	4 39	m	19 30	m	14 51	m
		+33		−43		−76
Sept. 1	5 12		18 47		13 35	
		+34		−48		−82
22	5 46		17 59		12 13	

This shows that the change in the length of the day in the last three weeks of August is not much less than that in the first three weeks of September, which are immediately followed by the equinox.

To examine the problem suggested more closely, with the simplifications admitted, the necessary equations are—

$$\sin D = \sin\omega \sin L$$
$$\cos T = -\tan\phi \tan D,$$

where ω is the obliquity of the ecliptic, ϕ the latitude of the place, and L, D, T are the Sun's longitude, declination, and hour angle at sunset. L is assumed to increase uniformly with the time, and 2T is a measure of the length of the day. Then

$$\cos D \cdot \frac{dD}{dL} = \sin\omega \cos L$$

$$\sin T \cdot \frac{dT}{dL} = \tan\phi \sec^3 D \sin\omega \cos L$$

$$\sin T \frac{dT}{dL}\frac{dD}{dL} = \tan\phi \sec^4 D(\sin^2\omega - \sin^2 D)$$

$$\sin^2 T\left(\frac{dT}{dL}\right)^2 = \tan^2\phi \sec^6 D(\sin^2\omega - \sin^2 D)$$

$$\frac{d^2T}{dL^2} = \frac{dT}{dL}\left(3 \tan D\frac{dD}{dL} - \cot T\frac{dT}{dL} - \tan L\right).$$

Hence

$$\begin{aligned}\sin^3 T\frac{d^2T}{dL^2} &= 3 \tan\phi \tan D \sec^4 D \sin^2 T(\sin^2\omega - \sin^2 D)\\ &\quad - \tan^2\phi \sec^6 D \cos T(\sin^2\omega - \sin^2 D)\\ &\quad - \tan\phi \tan D \sec^2 D \sin^2 T\\ &= -\sec^4 D \cos T(\sin^2\omega - \sin^2 D)(3 \sin^2 T + \tan^2\phi \sec^2 D)\\ &\quad + \sec^2 D \cos T \sin^2 T.\end{aligned}$$

A stationary value of $d\mathrm{T}/d\mathrm{L}$ is given by $\cos \mathrm{T} = 0$ or $\mathrm{D} = 0$, that is, at the equinoxes. Apart from this, similar values may also be given by the equation

$$\begin{aligned} 1 - \tan^2\phi \tan^2 \mathrm{D} &= (1 - \cos^2\omega \sec^2 \mathrm{D})(3 - 2\tan^2\phi \tan^2 \mathrm{D} + \tan^2\phi) \\ &= \sin^2\omega(3 + \tan^2\phi) + 2\cos^2\omega \tan^2\phi \tan^4 \mathrm{D} \\ &\quad - \tan^2 \mathrm{D}(3\cos^2\omega + \sin^2\omega \tan^2\phi + \tan^2\phi); \end{aligned}$$

or, as it may be written,

$$2\cos^2\omega \tan^2\phi(\tan^2\omega - \tan^2 \mathrm{D})^2 - 3(\sin^2\omega \tan^2\phi - \cos^2\omega)$$
$$(\tan^2\omega - \tan^2 \mathrm{D}) + \tan^2\omega \tan^2\phi - 1 = 0,$$

or again,

$$f(\mathrm{X}) = 2\mathrm{X}^2 - 3\mathrm{X}(1 - bc) + (1 - bc)(1 + b) = 0,$$

where

$$\mathrm{X} = 1 - \cot^2\omega \tan^2 \mathrm{D}, \qquad b = \cot^2\omega, \qquad c = \cot^2\phi.$$

Since $\mathrm{D} < \omega$, only positive roots of this quadratic in X possess any interest for the problem.

Two cases must now be distinguished :

(1) In the sub-arctic zone $bc = \cot^2\omega \cot^2\phi > 1$. Then the quadratic has one real positive and one real negative root.

Since

$$f(0) = (1 - bc)(1 + b) < 0$$
$$f(1) = b(1 + 2c - bc),$$

the necessary condition $1 > \mathrm{X} > 0$ is satisfied by the positive root only when $b < 2 + 1/c$, or

$$\cot^2\omega - 2 < \tan^2\phi < \cot^2\omega.$$

Were $\cot^2\omega < 2$ or $\omega > 35^\circ{\cdot}5$, this condition would be always satisfied. In our case $\cot\omega = 2{\cdot}3$ and $\tan^2\phi > 3{\cdot}29$. Hence, with the actual value of the obliquity of the ecliptic, it is only in the range of latitudes

$$61^\circ{\cdot}1 < \phi < 66^\circ{\cdot}5$$

that the rate of change in the length of the day becomes greatest at an intermediate date in each quarter, and is less at each equinox than before or after. In lower latitudes the greatest rate occurs at the equinox.

(2) In the arctic zone $bc < 1$, and the quadratic in X has two roots which are both positive if they are real. They are imaginary unless the discriminant

$$\Delta = (1 - bc)(1 - 9bc - 8b)$$

is positive. This requires $8b < 1$, $\tan^2\omega > 8$, $\omega > 70^\circ{\cdot}4$ for any latitude. Since the obliquity is far below this limit, the roots are imaginary, and the rate of change only becomes stationary at the equinox.

It is, however, possible to see that in arctic latitudes the quadratic in X cannot yield a valid root for any value of the obliquity. The Sun becomes circumpolar unless $\tan^2 \mathrm{D} < \cot^2\phi$, and therefore

$$1 > \mathrm{X} > 1 - bc.$$

But
$$f(1) = b(1 - bc + 2c) > 0$$
$$f(1 - bc) = b(1 - bc)(1 + c) > 0.$$

Hence the only possible range of X contains neither or both of the roots of $f(X) = 0$. But it cannot contain both, because their sum is $\frac{2}{2}(1 - bc)$. Hence it contains neither.

In the arctic zone, therefore, there is no turning-point in the rate of change except at the equinox. In both directions from the equinox, up to the point where the Sun becomes circumpolar and the 24-hour day or night supervenes, the rate of change in the length of the day increases.

The general result may be seen in another way. At the equinox $T = 90°$, $D = 0$, $L = 0$ or $180°$, and

$$dT/dL = \pm \tan \phi \sin \omega.$$

If the same rate obtains at any other time of year,

$$\sin T = \pm \sec^3 D \cos L,$$

and then
$$1 = \sec^6 D \cos^2 L + \tan^2 \phi \tan^2 D$$
$$\sin^2 \omega = \sec^6 D(\sin^2 \omega - \sin^2 D) + \tan^2 \phi \sin^2 \omega \tan^2 D;$$

or after division by $\tan^2 D \sin^2 \omega$,

$$g(\sec^2 D) = \cot^2 \omega \sec^4 D - \sec^2 D - \sec^2 \phi = 0.$$

This quadratic in $\sec^2 D$ has one real positive root, but it is also necessary that

$$1 < \sec^2 D < \operatorname{cosec}^2 \phi,$$

since the Sun must not be circumpolar. Now

$$g(1) = \cot^2 \omega - 2 - \tan^2 \phi$$
$$g(\operatorname{cosec}^2 \phi) = (\cot^2 \omega - \tan^2 \phi) \operatorname{cosec}^4 \phi,$$

and these expressions must have opposite signs. This can only happen when

$$\cot^2 \omega > \tan^2 \phi > \cot^2 \omega - 2,$$

or within the range of latitude found above. The rate of change at the equinoxes is then equalled at some intermediate date given by the equation $g(\sec^2 D) = 0$. Nearer the equator the same rate does not occur at any other time of year, because it is then a maximum, and similarly it does not occur again nearer the poles, because it is then a minimum.

That the length of the day is not necessarily changing most rapidly at the equinoxes is very clearly shown by the circumstances on the arctic circle itself ($\phi + \omega = 90°$). From a combination of the simple formulæ given above it follows that here

$$\frac{dT}{dL} = \pm \cos \omega \sec^2 D = \pm \frac{\cos \omega}{1 - \sin^2 \omega \sin^2 L},$$

which increases progressively from cos ω at the equinox to sec ω at the solstice. Since the mean effect of one day is to change L by 360°/365·25, and the effect of 1° in T is to change the length of the day by 8^m, the actual change is from $473^s \cos \omega = 7^m\ 14^s$ at the equinox to $473^s \sec \omega = 8^m\ 36^s$ at the solstice. An average daily rate of $7^m\ 53^s$ is required to change the 12-hour day into the 24-hour day (or night), and without direct examination of the point it could scarcely be anticipated that the actual rate of change is so nearly uniform through the year.

To take an example in an intermediate latitude, let $\phi = 63°{\cdot}4$, $\tan \phi = 2$. At the equinox the length of the day changes at the rate of $6^m\ 17^s{\cdot}2$. The maximum rate is given by X = 0·7941, D = 11° 8′, L = 29°, and therefore precedes and follows the equinox by about a month. This rate is $6^m\ 20^s{\cdot}2$, or only 3^s more than the intervening minimum at the equinox. The latter value of the rate occurs again when $\sec^2 D = 1{\cdot}071$, D = 14°·9, L = 40°·2, so that nearly six weeks before and after the equinox the length of the day is changing at the same rate as at the equinox itself. Thus in the latitude chosen, for a period of twelve weeks over the equinox, the length of the day changes at an almost uniform rate, the greatest rate differing from the least by only 3^s.

When the latitude 61°·1 is reached the two maxima coalesce with the minimum at the equinox, and in the latitude of England only the single maximum at the equinox remains. Here the maxima which have disappeared leave their trace only in the flattening of the equinoctial maximum, which is the feature remarked on at the beginning of this note.

From the astronomical point of view the question discussed above is of the most trivial. From another aspect it may not be without a certain importance. Mere duration of daylight is one of the factors on which the heat received at a given place from day to day depends. The seasonal change being given, it is reasonable to suppose that a steady rate of change will be a condition favourable to stable weather and unfavourable to violent and irregular disturbances. It is therefore interesting to observe that in an important zone, when the changes are most marked, the geometrical conditions conduce to some extent at least to secure this regularity in the meteorological order.

Errata.

Prof. Brown's paper.—P. 460, line 7, the sign of the second term on the right is minus.

P. 462, line 7, *for* test *read* last.

Pp. 462, 463, §§ 6, 8, place a bar over the letter p to distinguish it from the same letter used elsewhere in a different sense.

Dr. Reesinck's paper.—P. 505, line 6 from the bottom, *for* valid *read* wild.